

**MATH 166**  
**SUMMER 2011**  
**QUIZ 16**

1. (5 pt) Show by induction that

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}.$$

2. In this problem we consider the sequence

$$\left\{ \frac{1}{2}, \frac{1}{2} + \frac{1}{4}, \dots, \frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2^n}, \dots \right\}$$

- a) (5 pt) Show by induction that  $\frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2^n} = \frac{2^n - 1}{2^n}$   
b) (5 pt) Show that this sequence converges and find the limit.

## Formulae

- (1)  $\sin(2x) = 2 \sin(x) \cos(x)$
- (2)  $\cos(2x) = \cos^2(x) - \sin^2(x)$
- (3)  $\cos^2(x) = \frac{1}{2} + \frac{1}{2} \cos(2x)$
- (4)  $\sin^2(x) = \frac{1}{2} - \frac{1}{2} \cos(2x)$
- (5)  $\sin(A) \cos(B) = \frac{1}{2} [\sin(A - B) + \sin(A + B)]$
- (6)  $\sin(A) \sin(B) = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$
- (7)  $\cos(A) \cos(B) = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$
- (8)  $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
- (9)  $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
- (10)  $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
- (11)  $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
- (12)  $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
- (13)  $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
- (14)  $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$
- (15)  $S = \int_a^b 2\pi(x \text{ or } y) ds$
- (16)  $\int_{n+1}^{\infty} f(x) dx \leq R_n \leq \int_n^{\infty} f(x) dx$
- (17)  $\bar{x} = \frac{1}{A} \int_a^b x(f(x) - g(x)) dx$
- (18)  $\bar{y} = \frac{1}{2A} \int_a^b [(f(x))^2 - (g(x))^2] dx$
- (19)  $A = \int_a^b \frac{1}{2} r^2 d\theta$
- (20)  $\int \sec(x) dx = \ln |\sec(x) + \tan(x)| + c$
- (21)  $\int \sec^3(x) dx = \frac{1}{2} \sec(x) \tan(x) + \frac{1}{2} \ln |\sec(x) + \tan(x)| + c$
- (22) If  $t = \tan\left(\frac{x}{2}\right)$  then  $\sin(x) = \frac{2t}{t^2+1}$ ,  $\cos(x) = \frac{1-t^2}{t^2+1}$ ,  $dx = \frac{2dt}{t^2+1}$