

MATH 166
SUMMER 2011
QUIZ 29

1. For this problem, we will consider the differential equation

$$(x^2 + 1) \frac{dy}{dx} = y.$$

- a) (5 pt) Find a general solution for this equation.
- b) (5 pt) Find the limit of this solution as $x \rightarrow \pm\infty$.
- c) (5 pt) Find the specific solution if we are also given that $y(0) = 5$.

2. For this problem, recall that

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad -\infty < x < \infty.$$

- a) (5 pt) Find a series for $f(x) = e^{-x^2}$.
- b) (5 pt) Show that this series is a solution to the differential equation

$$\frac{dy}{dx} + 2xy = 0.$$

- c) (5 pt) Separate the equation (like in #1) and find a general solution.

Formulae

- (1) $\sin(2x) = 2 \sin(x) \cos(x)$
- (2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
- (3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2} \cos(2x)$
- (4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2} \cos(2x)$
- (5) $\sin(A) \cos(B) = \frac{1}{2} [\sin(A - B) + \sin(A + B)]$
- (6) $\sin(A) \sin(B) = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$
- (7) $\cos(A) \cos(B) = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$
- (8) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
- (9) $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
- (10) $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
- (11) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
- (12) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
- (13) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
- (14) $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$
- (15) $S = \int_a^b 2\pi(x \text{ or } y) ds$
- (16) $\int_{n+1}^{\infty} f(x) dx \leq R_n \leq \int_n^{\infty} f(x) dx$
- (17) $\bar{x} = \frac{1}{A} \int_a^b x(f(x) - g(x)) dx$
- (18) $\bar{y} = \frac{1}{2A} \int_a^b [(f(x))^2 - (g(x))^2] dx$
- (19) $A = \int_a^b \frac{1}{2} r^2 d\theta$
- (20) $\int \sec(x) dx = \ln |\sec(x) + \tan(x)| + c$
- (21) $\int \sec^3(x) dx = \frac{1}{2} \sec(x) \tan(x) + \frac{1}{2} \ln |\sec(x) + \tan(x)| + c$