MATH 166
SPRING 2007
FINAL EXAM

1. (32 pt) Evaluate the following integrals.

$$\text{a)} \int \sqrt{x}e^{\sqrt{x}}\, dx, \qquad \text{b)} \int \frac{\cos(x)}{\sin^3(x)+\sin(x)}\, dx \qquad \text{c)} \int_2^\infty \frac{dx}{x^2\sqrt{x^2+4}}$$

$$\text{d)} \int_0^{\sqrt[4]{3}} 2x\tan^{-1}(x^2)\, dx$$

2. (6 pt) Determine if the following sequences converge or diverge.

$$\text{a)} \left\{(-1)^n n\sin\left(\frac{1}{n}\right)\right\}_{n=1}^{\infty} \qquad \text{b)} \left\{2^{-s_n}\right\}_{n=1}^{\infty} \text{ where } s_n \text{ is the } n^{\text{th}} \text{ partial sum of a positive term series.}$$

3. (15 pt) Determine if the following series converge or diverge.

$$\text{a)} \sum_{n=1}^{\infty}(-1)^n\frac{n}{2n^2+1} \qquad \text{b)} \sum_{n=2}^{\infty} ne^{-n} \qquad \text{c)} \sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}$$

4. (15 pt) Consider the curve given by the parametric equations $x = \frac{2}{t^2+1}$ and $y = \frac{2t}{t^2+1}$. For your convenience, $\frac{dx}{dt} = \frac{-4t}{(t^2+1)^2}$ and $\frac{dy}{dt} = \frac{2-2t^2}{(t^2+1)^2}$.

a) Sketch this curve, indicating the direction in which the curve is drawn as t increases.
b) Find the total area enclosed by this curve.
c) Find the total length of this curve.

5. (12 pt) Consider the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; a, b > 0$.

a) Find the area enclosed by the entire ellipse.
b) Locate the $y-$coordinate of the centroid of the upper half of this ellipse.
c) Find the volume obtained when the upper half of this ellipse is revolved about the $x-$axis.

6. (6 pt) Consider the polar curve $r = \tan(\frac{1}{2}\theta); 0 \le \theta < \pi$.

a) Sketch this polar curve.
b) Find the area of the region in the first quadrant that is bounded by this curve, the $x-$axis, and the $y-$axis.

7. (6 pt) Find the Maclaurin series for $f(x) = e^{-x^2}$ and use this series to approximate $\int_0^1 x^2e^{-x^2}\, dx$ with error less than $\frac{1}{50}$.

8. (6 pt) A square window of side length R has its top border D feet below the surface of the ocean. If water weighs 62.5 pounds per cubic foot and pressure at depth x is given by $62.5x$, find the force due to hydrostatic pressure on the window (recall that $F = PA$).

9. (6 pt) Let $f(x)$ be a continuous function.

a) Show that if $f(x)$ is odd, then the average value of $f(x)$ on $[-a, a]$ is 0.
b) Show that if $f(x)$ is even, then the average value of $f(x)$ on $[-a, a]$ is equal to the average value of $f(x)$ on $[0, a]$.

10. (6 pt) Solve the differential equation $\frac{dy}{dx} = \frac{y}{x^2+1}$ given that $y(0) = 2$.