MATH 166
SPRING 2008
FINAL EXAM

1. (32 pt) Evaluate the following integrals.

$$\text{a)} \int e^{2x}\sin(e^x)\,dx, \qquad \text{b)} \int \frac{4f'(x)}{(f(x))^2((f(x))^2+4)}\,dx \qquad \text{c)} \int_0^1 x\ln(x)\,dx$$

$$\text{d)} \int \frac{1}{\sqrt{x^2-2x}}\,dx$$

2. (15 pt) Determine if the following series converge or diverge.

$$\text{a)} \sum_{n=1}^{\infty} \frac{\sqrt{n^3+1}}{\sqrt[3]{4n^8+n^6+9}} \qquad \text{b)} \sum_{n=2}^{\infty} (-1)^n \frac{1}{\ln(\ln(n))} \qquad \text{c)} \sum_{n=1}^{\infty} \frac{(2n^2+1)^{3n}}{(3n^3+4)^{2n}}$$

3. (7 pt) Set up and evaluate an integral for finding the surface area of a sphere of radius R.

4. (10 pt) Consider the parametric equations $x = t^2 - 12t$ and $y = t^3 - 12t$.
   a) Sketch this curve.
   b) This curve intersects the $x-$axis three times. Set up the integral to find the region bounded by this curve and the $x-$axis between these three points of intersection (you do not need to evaluate the integral).

5. (6 pt) Consider the polar curve $r = 1 + \sin(3\theta)$.
   a) Sketch this polar curve.
   b) Find the total area enclosed by this curve.

6. (15 pt) Consider the region bounded by $f(x) = \sin(x)$, $g(x) = \sin(2x)$, $y = 0$, $x = 0$, and $x = a$ where a is the smallest positive x value where $f(x)$ and $g(x)$ intersect.
   a) Find the area of this region.
   b) Locate the centroid of this region.
   c) Find the volume obtained when this region is revolved about the line $y = 2$.

7. (6 pt) Evaluate

$$\int_0^1 \frac{dx}{x^4+16}$$

with error less than $\frac{1}{10000}$.

8. (6 pt) Find the center, radius, and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{\ln(n)(3x-6)^{2n}}{n9^n}.$$

9. (8 pt) A cylindrical barrel with base radius R and height h is filled with a liquid of density ρ to a depth D where $0 < D \leq h$.
   a) Find the force due to hydrostatic pressure on the sides.
   b) Find the work done in pumping all the liquid out of the barrel.

10. (5 pt) Solve the differential equation $x\sin(y)\frac{dy}{dx} = \ln(x)\cos(y)$.