MATH 166
SPRING 2011
FINAL EXAM

1. (32 pt) Evaluate the following integrals.

$$\text{a)} \int x\ln(x)\,dx, \qquad \text{b)} \int e^{3x}\sin(x)\,dx \qquad \text{c)} \int_0^1 \sqrt{4x^2+1}\,dx$$
$$\text{d)} \int \frac{x^2+2x-1}{x^3-x}\,dx$$

2. (18 pt) Determine if the following series converge or diverge.

$$\text{a)} \sum_{n=1}^{\infty}(-1)^n\frac{\ln(n^2)}{n} \qquad \text{b)} \sum_{n=2}^{\infty}(\frac{n+1}{1-2n})^{2n} \qquad \text{c)} \sum_{n=1}^{\infty} n\sin(\frac{1}{n^3})$$

3. (10 pt) Determine if the following sequences converge or diverge.

$$\text{a)}\{2n\tan(\frac{1}{3n})\} \qquad \text{b)}\{\sqrt{5}, \sqrt{5+\sqrt{5}}, \sqrt{5+\sqrt{5+\sqrt{5}}}, \cdots\}$$

4. (12 pt) Consider the parametric equations $x = t^2 - 2\ln(|t|)$ and $y = t^3 - 3t$.
   a) Find the derivative $\frac{dy}{dx}$. What happens near $t = \pm 1$?
   b) Sketch this curve.
   c) What is the area under the portion of the curve corresponding to $-1 \le t < 0$?

5. (12 pt) Consider the polar curve $r^2 = \sin(\theta)$.
   a) Sketch this polar curve (be careful).
   b) Write down (but do not evaluate) an integral that represents the length of the upper loop of this curve.
   c) Find the area enclosed by the upper loop of this curve.
   d) Does this curve consist of two circles? Why or why not?

6. (12 pt) Consider the region bounded by $f(x) = mx, m > 0$ and $g(x) = x^2$.
   a) Find the area of this region.
   b) Locate the centroid of this region.
   c) Find the volume obtained when this region is revolved about the $y-$axis.
   d) Find the volume obtained when this region is revolved about the line $y = -2$.

7. (5 pt) Evaluate

$$\int_0^1 e^{-x^2}dx$$

with error less than $\frac{1}{1000}$.

8. (6 pt) Consider the curve

$$f(x) = \int_0^x \sqrt{e^{2t}-1}\,dt.$$

   a) Find the length of the curve for $0 \le x \le a$.
   b) Find the surface area obtained when this curve ($0 \le x \le a$) is revolved about the $y-$axis.

9. (3 pt) Solve the differential equation $\frac{dy}{dx} = (1+x^2)(1+y^2)$ if $y(0) = 1$.

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$
(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
(5) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
(6) $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
(7) $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
(8) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
(9) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
(10) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
(11) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2}d\theta$
(12) $S = \int_a^b 2\pi(x \text{ or } y)ds$
(13) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
(14) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$
(15) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2]dx$
(16) $A = \int_a^b \frac{1}{2}r^2 d\theta$
(17) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$
(18) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$