**MATH 166**
**SPRING 2012**
**FINAL EXAM**

1. (32 pt) Evaluate the following integrals.

$$\text{a)} \int_0^2 \frac{dx}{\sqrt{4x^2+9}} \qquad \text{b)} \int e^{\sqrt[3]{x}}\, dx \qquad \text{c)} \int \frac{2x^3-4}{x^4+4x^2}\, dx$$
$$\text{d)} \int_0^\infty \frac{2x}{x^4+1}\, dx$$

2. (18 pt) Determine if the following series converge or diverge.

$$\text{a)} \sum_{n=1}^{\infty} (-1)^n \frac{(n!)^2}{(2n)!} \qquad \text{b)} \sum_{n=2}^{\infty} \frac{\ln(n)}{n^2} \qquad \text{c)} \sum_{n=1}^{\infty} (-1)^n \frac{n-\sin(n)}{980n+7}$$

3. (10 pt) Determine if the following sequences converge or diverge.

$$\text{a)}\{n^2 \sin(\frac{3}{n}) \tan(\frac{2}{n})\} \qquad \text{b)}\{1, 1+\frac{1}{2}, 1+\frac{1}{2}+\frac{1}{3}, \cdots, 1+\frac{1}{2}+\cdots+\frac{1}{n}, \cdots\}$$

4. (12 pt) Consider the parametric equations $x = t^3 - 12t$ and $y = t^3 - 3t$.
   a) Sketch this curve.
   b) For what value(s) of $t$ does this curve intersect the $x-$axis?
   c) Find the area bounded by this curve and the $x-$axis.

5. (12 pt) Consider the polar curve $r = 1 + \sin(\frac{\theta}{2})$.
   a) Sketch this polar curve.
   b) Find the area enclosed by the inner loop of this curve.

6. (8 pt) Consider a cylindrical tank of height $h$ and radius $R$ filled with a liquid of density $\rho$
   a) Find the force due to hydrostatic pressure on the side of the tank.
   b) How much work is done in pumping all the liquid out of the tank through a spigot that is $d$ unitis above the top of the tank.

7. (6 pt) Find an infinite series for $\tan^{-1}(x)$ and use this to find a series for $\frac{\pi}{4}$. How many terms are needed to guarantee that the approximation $s \approx s_n$ has error no more than $\frac{1}{1000}$.

8. (8 pt) Consider the curve

$$f(x) = \ln(|\sec(x)|).$$

   a) Find the length of the curve for $0 \le x \le \frac{\pi}{4}$.
   b) Set up the integral that gives the surface area obtained when this curve is revolved about the line $y = -2$.

9. (4 pt) Solve the differential equation $y' \sin(x) = (y^2+1)\cos(x)$.

Formulae

(1) $\sin(2x) = 2\sin(x)\cos(x)$
(2) $\cos(2x) = \cos^2(x) - \sin^2(x)$
(3) $\cos^2(x) = \frac{1}{2} + \frac{1}{2}\cos(2x)$
(4) $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$
(5) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
(6) $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$
(7) $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$
(8) $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
(9) $|E_T| \leq \frac{K(b-a)^3}{12n^2}$
(10) $|E_S| \leq \frac{K(b-a)^5}{180n^4}$
(11) $L = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}dx = \int_a^b \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}dt = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2}d\theta$
(12) $S = \int_a^b 2\pi(x \text{ or } y)ds$
(13) $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_n^{\infty} f(x)dx$
(14) $\overline{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx$
(15) $\overline{y} = \frac{1}{2A}\int_a^b [(f(x))^2 - (g(x))^2]dx$
(16) $A = \int_a^b \frac{1}{2}r^2 d\theta$
(17) $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + c$
(18) $\int \sec^3(x)dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + c$