MATH 166
SUMMER 2012
FINAL EXAM

1. (32 pt) Evaluate the following integrals.

a) $\int \frac{x^2}{\sqrt{9-4x^2}}dx$ b) $\int e^{2x}e^{e^x}\ dx$ c) $\int \frac{\cos(x)}{\sin^2(x)(\sin^2(x)+1)}\ dx$

d) $\int_0^1 (\ln(x))^2\ dx$

2. (16 pt) Determine if the following series converge or diverge.

a) $\sum_{n=1}^{\infty} \frac{\sqrt{n^2+1}}{\sqrt[3]{n^7+n^4+1}}$ b) $\sum_{n=2}^{\infty} \sin(\frac{1}{n})$

3. (18 pt) Consider the region in the first quadrant bounded by $y = x^n,\ n > 1$, the $y-$axis and the line $y = 1$.
   a) Find the area of this region.
   b) Find the $x-$coordinate of the centroid of this region.
   c) Find the $y-$coordinate of the centroid of this region.
   d) Find the volume obtained when this region is revolved about the $x-$axis.
   e) Find the volume obtained when this region is revolved about the $y-$axis.
   f) What happens (should happen) to the centroid as $n \longrightarrow \infty$?

4. (12 pt) Consider the parametric equations $x = t^3 - 3t$ and $y = \frac{t^2}{t^2+1}$.
   a) Compute $\frac{dx}{dt}$ and $\frac{dy}{dt}$ and find where x and y are increasing and decreasing.
   b) Find the values of t for which this curve intersects the $y-$axis.
   c) Sketch this curve.
   d) Find the area enclosed by the loop in this curve.

5. (8 pt) Consider the polar curve $r = a + \sin(\theta),\ a > 1$.
   a) Sketch this polar curve.
   b) Calculate the area enclosed by this curve.

6. (8 pt) Consider a cylindrical bucket of height h and radius R is filled with a fluid of density ρ.
   a) Find the force due to hydrostatic pressure on the side of the bucket.
   b) If this bucket weighs b units and is drawn up a well that is w units deep by a cable that weighs c per unit length, find the work done.

7. (8 pt) Consider the sequence defined by $a_1 = 4$ and $a_{n+1} = \sqrt{\frac{1}{2}a_n^2 + 1},\ n \geq 1$.
   a) Show that each term in this sequence is between 0 and 4.
   b) Show that this sequence is decreasing.
   c) Explain why this sequence converges.
   d) What is the limit of this sequence?

8. (8 pt) Consider the power series

$$\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{2^n}.$$

   a) Find the center, radius, and interval of convergence of the power series.
   b) If $f(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{2^n}$ then find $\int_{-1}^{1} f(x)dx$ with error no more than $\frac{1}{100}$.