MATH 421-621
SPRING 2013
HOMEWORK 11

Due Wednesday May 1, 2013.

1. (5 pt) Let F be a field and let $\overline{F}$ be its algebraic closure. Show that $\overline{F}$ is algebraically closed.

2. (5 pt) Show that any field has an algebraic closure.

3. Let F be a field and $f(x) \in F[x]$ be an irreducible polynomial and K the splitting field of $f(x)$ over F.

a) (5 pt) Show that α is a multiple root in K if and only if α is a root of $f'(x)$.
b) (5 pt) If $\mathrm{char}(F) = 0$ then show that $f(x)$ is a separable polynomial.
c) (5 pt) Show that if F is a finite field, then $f(x)$ is separable.
d) (5 pt) Show that for all n, the polynomial $x^{p^n} - x$ is separable over $\mathbb{Z}_p$.
e) (5 pt) Show that the set of roots of $x^{p^n} - x$ forms a field extension of $\mathbb{Z}_p$.
f) (5 pt) Show that for all n there is a unique (up to isomorphism) field of p^n elements.