MATH 421-621
SPRING 2013
HOMEWORK 4

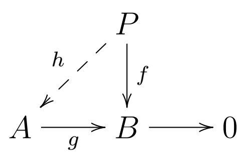
Due Wednesday, February 6, 2013.

1. (5 pt) Let F be a free $R-$module with basis $\{e_i\}_{i\in\Lambda}$, and M another $R-$module. If $\{x_i\}_{i\in\Lambda}$ is a collection of elements of M, show that there is a unique homomorphism $\phi : F \longrightarrow M$ such that $\phi(e_i) = x_i$ for all $i \in \Lambda$.

2. We say that the $R-$module P is *projective* if there is a free $R-$module F and another $R-$module K such that $P \oplus K \cong F$.

a) (5 pt) Show that if F is free then it is projective.
b) (5 pt) Explain why the module K is also projective.
c) (5 pt) Show that $\mathbb{Q}$ is not a projective $\mathbb{Z}-$module,
d) (5 pt) Give an example of a projective $R-$module that is not free.

3. Consider the following diagram of $R-$modules (the homomorphism g is surjective).



a) (5 pt) Show that if P is free, then there is an $R-$module homomorphism $h : P \longrightarrow A$ such that $gh = f$.
b) (5 pt) Show that if P is projective, then there is an $R-$module homomorphism $h : P \longrightarrow A$ such that $gh = f$.

4. (5 pt) Show that if P and Q are projective $R-$modules, then $P \otimes_R Q$ is also projective.