MATH 421-621
SPRING 2013
HOMEWORK 7

Due Wednesday April 3, 2013.

1. Find the canonical forms (the rational canonical form, primary rational canonical form and Jordan canonical form if possible) for the following matrices over $\mathbb{Q}$:

a) (15 pt) $\begin{bmatrix} -1 & 2 & -1 & 0 \\ -2 & 3 & -1 & 0 \\ 1 & -1 & 2 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix}$

b) (15 pt) $\begin{bmatrix} 3 & 1 & 0 & 1 & 1 \\ 0 & 3 & 0 & -1 & 0 \\ 0 & -2 & 4 & 2 & 0 \\ 0 & -1 & 0 & 3 & 0 \\ 1 & -1 & 0 & -1 & 3 \end{bmatrix}$

3. A matrix A is said to be nilpotent if there is an $m \geq 1$ such that $A^m = 0$. Additionally, we define the trace of A ($\mathrm{tr}(A)$) to be the sum of the diagonal elements of A. For this problem, you may assume that A is an $n \times n$ matrix over a field $\mathbb{F}$.

a) (5 pt) Show that $\mathrm{tr}(AB) = \mathrm{tr}(BA)$.
b) (5 pt) Show that if P is an invertible $n \times n$ matrix then $\mathrm{tr}(P^{-1}AP) = \mathrm{tr}(A)$.
c) (5 pt) Show that A is nilpotent if and only if all of its eigenvalues are 0.
d) (5 pt) Show that if A is nilpotent, then $\mathrm{tr}(A) = 0$.
e) (5 pt) Determine the status of the converse of the statement in part d).