MATH 772
SUMMER 2006
HOMEWORK 5

Due Wednesday, August 2, 2006 (HA HA).

1. (5 pt) Show that if $\mathbb{F}$ is a field such that $\mathrm{char}(\mathbb{F}) \neq 2$, then there is a one to one correspondence between the set of quadratic extensions of $\mathbb{F}$ and the nontrivial elements of the group $\mathbb{F}^*/(\mathbb{F}^*)^2$.

2. For the following list of fields, find the number of quadratic extensions.
   a) (5 pt) K where K is any algebraic number field.
   b) (5 pt) $\mathbb{F}$ where $\mathbb{F}$ is any finite field.
   c) (5 pt) $\mathbb{R}$.
   d) (5 pt) $\mathbb{C}$.
   e) (5 pt) $\mathbb{Q}_p$ where p is an odd prime.
   f) (5 pt) $\mathbb{Q}_2$.

3. (5 pt) Is there a countable field of characteristic 0 that possesses a unique quadratic extension (if so, give an example and if not prove that one cannot exist)?

4. (5 pt) Show that the integers $\mathbb{Z}$ form a dense subset (with respect to the $p-$adic metric) of $\mathbb{Z}_p$. What is $\overline{\mathbb{Z}} \bigcap \mathbb{Q}$ (where $\overline{\mathbb{Z}}$ is the closure of $\mathbb{Z}$ with respect to the $p-$adic metric)?

My summer begins when this homework ends...-C. Hashbarger