Junior Colloquium

Department of Mathematics
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1 Spring 2015

Speaker: Azer Akhmedov  
Title: An excursion to Knot Theory  
Date: April 23  
Abstract: Knots are obtained by embedding a circle in $\mathbb{R}^3$. Pictures and handicrafts of knots have appeared in ancient Chinese art several hundred years B.C. The first systematic study of knots though started in the 19th century by a Scottish physicist Peter Guthrie Tait who classified all knots to 10 crossings and made several important conjectures about knots (the most recent of these conjectures is proven in 1991). In my talk, I will recall some elementary notions of knots (such as prime knots, torus knots, fibered knots, pretzel knots, alternating knots, satellite knots, etc.). Nowadays, knots are interesting objects not just to mathematicians, but also to physicists, biologists, chemists, etc. In my talk, I will recall some elementary notions of knots (such as prime knots, torus knots, fibered knots, pretzel knots, alternating knots, satellite knots, etc.). Many, many examples and pictures will be provided. I’ll also briefly mention several well known knot invariants, but concentrate mostly on computing the Alexander polynomial of knots.

Speaker: Liz Sattler  
Title: Hausdorff dimension of sub-fractals associated with sub-shifts  
Date: February 26  
Abstract: Many fractals can be defined by an iteration function system (IFS) consisting of finitely many maps. We can associate a letter to each map and use a symbolic space to describe the fractal. One way to define a sub-fracta is to relate the IFS to a sub-shift of finite type (SFT) on the symbolic space. In this talk, we will formally describe the connection between such sub-fractals and SFTs. We will also discuss some techniques for calculating the Hausdorff dimension of these sets.
Speaker: Sean Sather-Wagstaff
Title: Putting the Ext in Extensions
Date: February 19

Abstract: Short exact sequences (a.k.a., "extensions") are fundamental in algebra. For instance, one uses these to prove the existence of rational canonical forms in linear algebra. In this talk I will discuss how the cohomological construction “Ext” controls the behavior of extensions in surprisingly deep ways.

Speaker: Semere Habtemicael
Title: BN-S model for valuing variance, volatility and covariance swap
Date: February 12

Abstract: In this talk we are going to discuss the pricing of one of the most popular financial derivatives which is called "swap". Swap is a financial derivative in which two counter parties exchange financial instruments such as cash flow. We use non-Gaussian Ornstein-Uhlenbeck process driven by Levy subordinator to model the dynamics of stock price (BN-S model) and use it to value the price of variance, volatility, and covariance swap.
2 Fall 2014

Speaker: Sean Sather-Wagstaff  
Title: Undecidable questions in commutative algebra  
Date: December 4  
Abstract: Construction of a projective resolution of a module is the process of iteratively approximating the module by projective modules. It is natural to ask when this process terminates. I will discuss work of Barbara Osofsky showing that the answer depends on whether or not you assume that the Continuum Hypothesis holds.

Speaker: Maria Alfonseca Cubero.  
Title: If you can hide behind it, can you hide inside it?  
Date: November 6  
Abstract: Let K and L be two convex bodies containing the origin. If for each hyperplane passing through the origin, the projection of K can be rotated around the origin to fit inside the projection of L. Can K be rotated to fit into L? We will present counterexamples to this question, and additional conditions on K and L that make the result true.

Speaker: Tom Dunn  
Title: Some Properties of Monomial Ideals  
Date: October 23  
Abstract: Properties of a monomial ideal can be seen in a lattice diagram representing a monomial ideal I in a polynomial ring R. Such properties include whether I is m-primary. If it is, we can relatively easily calculate the length of modules of the form $R/I^n$ and $I^{n-1}/I^n$, integral closure, the Hilber-Samuel multiplicity, and obtain information about reductions. If I is not m-primary, we can calculate the j-multiplicity.
Speaker: Caroline Turnage-Butterbaugh  
Title: Moments of products of automorphic L-functions  
Date: October 16

Abstract: The Riemann zeta-function $\zeta(s)$ and its generalizations, L-functions, are ubiquitous yet mysterious functions in number theory. These functions can be defined in association with a plethora of mathematical objects, including Dirichlet characters, number fields, and modular forms. We will begin with an introduction to the Riemann zeta-function and motivate the study of its moments. We will then consider general L-functions, specifically arbitrary products of L-functions attached to irreducible cuspidal automorphic representations of $GL(m)$ over $\mathbb{Q}$. The Langlands program suggests essentially all L-functions are of this form. Assuming some standard conjectures, I will discuss how to estimate two types of moments: the continuous moment of an arbitrary product of primitive L-functions and the discrete moment (taken over fundamental discriminants) of an arbitrary product of primitive L-functions twisted by quadratic Dirichlet characters. This is a generalization of results of K. Soundararajan and is inspired by the work of V. Chandee.

Speaker: Aaron Feickert  
Title: Resolutions and semidualizing modules  
Date: October 9

Abstract: Projective and injective modules are of key importance in algebra, in part because of their useful homological properties. The notion of C-projective and C-injective modules generalizes these constructions. In particular, these modules may be used to construct resolutions and define related homological dimensions in a natural way. When C is a semidualizing module, the C-projective and C-injective modules have particularly useful homological properties. Further, one may combine projective and C-projective resolutions to construct complete PC-resolutions (and, dually, complete IC-resolutions) that yield other modules with nice homological properties. We will survey these constructions. No prior knowledge of homological algebra is assumed.
Abstract: In 2000, Viswanath published a paper that used technology to give a specific example of a general result due to Furstenberg and Kesten (1960). The specific example was that if one took the standard Fibonacci sequence, where the nth term was obtained by addition of the previous two terms (1, 1, 2, 3, 5, 8...), and instead of adding each time, flipped a fair coin that decided whether one adds or subtracts. It is a well known fact that the quotient of consecutive Fibonacci numbers converges to the Golden Ratio as n approaches infinity, and Furstenberg and Kesten proved that the random Fibonacci sequence would almost always have a similar convergence property. Viswanath gave the initial digits of the new mathematical constant, (1.13....). In this talk, we will discuss a subset of the truly random Fibonacci sequences: the periodic Fibonacci sequences. The subset has measure 0, but there are still many interesting properties that they possess. If time permits, we will also discuss similar properties for the tribonacci sequence, and the n-nacci sequence in general.
3  Spring 2014

Speaker: Jessica Striker  
Title: Poset and polytope perspectives on alternating sign matrices (a thesis story)  
Date: May 8  
Abstract: In this talk, I’ll tell you about my thesis work in narrative form, including the stories behind the various results. I’ll also include some tips for getting through grad school and learning to do research.

Speaker: Melissa Duchsherer  
Title: Derivations of Semicrossed Products  
Date: May 1  
Abstract: We will consider a particular operator algebra, called a semicrossed product, generated by a symbol and continuous functions on a compact Hausdorff space. We will then define point derivations and discuss their basic properties and what it means for a point derivation to be inner. Finally, we will focus on a particular representation of a semicrossed product into an upper triangular matrix, and we will show that every point derivation associated with this representation is inner. A passing acquaintance with Math 750 and matrices is all that is required.

Speaker: Zheng Yang (University of Nebraska Lincoln)  
Title: Connected sums of k-algebras  
Date: April 24  
Abstract: A connected sum is a construction that can be used to construct Gorenstein rings. I will discuss a characterization of connected sums of k-algebras, and explore some relations between connected sums and associated graded rings, based on joint work with H. Ananth and E. Celikbas.
Speaker: Liz Sattler
Title: Finding the Hausdorff dimension of the canopy of a tree fractal I & II
Dates: April 3, 10

Abstract: In this talk, we will discuss the construction of tree fractals and canopies. The fractal dimension is a useful way to distinguish two fractals. We’ll focus on the Hausdorff dimension and will discuss different methods for calculation. We will focus on one calculation method, which involves defining a topological pressure and finding a Borel probability measure that is supported by the canopy. We will use this method to find the Hausdorff dimension of some well known fractals.

Speaker: Erin Brownlee
Title: Graph Algebras
Dates: February 20, 27

Abstract: This will be a two-part talk; the first talk is meant to be a refresher on (or brief introduction to) the necessary basic operator theory and $C^*$-algebras. This is meant for those who have not seen these before or for whom it has been a while since working in these topics. The second talk will be an introduction to Graph Algebras; this is a study of certain families of operators on (generally finite) directed graphs, and the $C^*$-algebras these families generate. We will explore the basic properties of these, as well as some interesting known results about them.
4 Fall 2013

Speaker: Corey Vorland  
Title: Random Numbers for a Few Systems of Inequalities  
Date: December 5

Abstract: The branch of mathematics known as Ramsey Theory is the study of minimal conditions for a particular structure to appear. On the integers, we consider the following problem: Given a linear equation, inequality, or system of equations/inequalities $L$, find the least integer $n$ such that when we color the ordered set $(1, 2, ..., n)$ using $t$ colors, we are guaranteed a solution to $L$ in $(1, 2, ..., n)$ in one color. In this talk, we will look at this problem using various choices for $L$. This talk will be accessible to both graduate and undergraduate students as no prior knowledge is needed.

Speaker: Mark Spanier  
Title: The Beurling-Selberg Extremal Problem and Applications  
Date: October 31

Abstract: The Beurling-Selberg Extremal Problem looks at finding optimal entire functions of a given exponential type that majorize or minorize a real valued function while minimizing the $L_1$-norm of the difference. Often the existence of such functions lead to optimal bounds in certain inequalities. In this talk, I will describe some classical applications of such problems and then present some results and generalizations for the Poisson and conjugate Poisson kernels. This talk is intended to be accessible to all graduate students and advanced undergraduates as I will describe any needed results from Complex and Fourier Analysis.
Speaker: Artem Novozhilov  
Title: Around Hilbert’s 16th problem  
Date: November 14

Abstract: Differential equations are sometimes considered (especially within the US system of higher education) as something which is taught right after the calculus, with a clear engineering taste and a lot of tedious and uninsightful calculations. Upper level courses in differential equations often have at least some applied flavor. These facts notwithstanding, the mathematical theory of differential equations is an extremely interesting and rich area on its own, which gave birth to such subjects as topology and Lie group theory, to mention just a few, and which is in the center of current mathematical activity. The modern theory of differential equations can be roughly divided into two parts: Qualitative theory of differential equations and Dynamical systems theory. The former deals mostly with analytical (or even polynomial) systems on the plane and can be described as relatively ”orderly world,” whereas the latter deals with flows in dimensions greater than two, and can be called ”the realm of chaos.” In this presentation I would like to talk about the qualitative theory of differential equations, which centered during approximately last hundred years around the 16th Hilbert’s problem. This problem (to be precise, the second part of it) can be stated as: What can be said about the number and location of limit cycles of a planar polynomial vector field of degree n? This problem is still open in its full generality, and I will discuss what is actually known today about it.

Speaker: Brian Chapman  
Title: Adic Transformations and the Pascal Adic  
Date: October 24

Abstract: Given an infinite, directed, graded graph G, an adic transformation is a dynamical system on the space of infinite paths of G. We describe some well-known dynamical systems and give their equivalent adic transformations. In particular, we discuss some dynamical properties of an adic transformation whose underlying graph is related to Pascal’s triangle.
Speaker: Dianna Kennedy  
Title: "Neat" Rings  
Date: October 17  

Abstract: Recall that an element $x$ in a ring $R$ is idempotent if $x^2 = x$. We’ll use idempotents in a discussion about clean rings and neat rings.

Speaker: Hannah Altmann  
Title: Semidualizing modules (and complexes) over tensor products  
Date: October 3  

Abstract: Let $R$ be a commutative, noetherian ring with identity. An $R$-module $C$ is semidualizing if $C$ is finitely generated over $R$, the homothety map $X_C^R: R \to \text{Hom}_R(C, C)$ is an isomorphism, and $\text{Ext}_R^i(C, C) = 0$ for all $i > 0$. We discuss the existence of nontrivial semidualizing modules (and complexes) over tensor products.

Speaker: Thomas Dunn  
Title: Modules of Finite Length and the Hilbert Function  
Date: September 26  

Abstract: First, we will define what it means for a module to be Artinian and under what conditions the module $R/I$ ($R$ a commutative ring with identity and $I$ an ideal) has finite length. The associated graded ring and the Hilbert function will be defined and we will prove that the Hilbert function eventually becomes a polynomial. From this Hilbert polynomial, we will define the multiplicity $e(I)$ and mention a few theorems regarding it.

Speaker: Pye Aung  
Title: Ring Structures on $R \oplus E$ and Gorenstein Injectivity  
Date: September 19  

Abstract: When $E$ is a semidualizing module over a commutative ring, Holm and Jrgensen studied some connections between $E$-Gorenstein injectivity/projectivity and Nagata’s trivial extension. We generalized some of those results to other general constructions including D’Anna and Fontana’s amalgamated duplication of a ring along an ideal and Esescu’s Pseudocanonical covers. We also identified some key properties of those general constructions that enable their connections to Gorenstein injectivity and projectivity.
Speaker: Aaron Feickert  
Title: Cracking the Enigma (I,II)  
Dates: September 5,12  

Abstract: The Enigma cipher machine was used extensively by German military forces during the second World War to send and receive encrypted messages. Though it was thought to be unbreakable, several flaws in the machine’s operation led to breakthroughs by Polish and British cryptanalysts that allowed Allied forces to decrypt messages and shortened the length of the war by several years. In the first part of this talk, we will investigate the Enigma machine’s operation, make working paper simulators, and discuss the combinatorics that gave the machine its reputation as an ”unbreakable” cipher. In the second part, we will develop the mathematical tools used by Polish mathematicians to break the cipher, and work through a complete message crack. If time permits, we will also discuss the British role in the Enigma crack, paying special attention to the role of Alan Turing in ending the war. This talk requires only a basic understanding of combinatorics, permutation theory, and the operation of scissors and tape, and will be accessible to all graduate students and advanced undergraduates.
5 Spring 2013

Speaker: Aaron Feickert
Title: Public-key cryptography
Date: May 3

Abstract: Public-key cryptography provides a unique and fascinating solution to problems involving authentication and privacy of information. The idea is to use a pair of encryption keys instead of a single key used by all parties. We’ll explore the underlying mathematics and applications.

Speaker: Artem Novozhilov
Title: Modeling epidemics on random networks
Date: April 26

Abstract: I am going to talk about very basic models in mathematical epidemiology, mathematical insights that can be obtained from these simple models, shortcomings of these models, and about the connections with random network theory, which has recently become a very powerful tool in mathematical epidemiology (and in various other fields of current research). In a more general context, my talk can be considered as a promotion of Fall 2013 graduate course Math 767: Topics in Applied Mathematics: Mathematics of Networks; I will try to present a wide picture of what I am going to discuss in this course, and what kind of mathematics can be learned.

Speaker: Rich Wicklein
Title: Codualizing modules and complexes
Date: April 12

Abstract: Let R be a commutative noetherian ring. A finitely generated R-module C is said to be semidualizing if $\text{Ext}_R^i(C, C) = 0$ for all $i > 0$ and $R \cong \text{Hom}_R(C, C)$. When R is local, an artinian R-module T is said to be quasidualizing if $\text{Ext}_R^i(T, T) = 0$ for all $i > 0$ and $\hat{R} \cong \text{Hom}_R(T, T)$. Using the notion of I-cofiniteness, we introduce a unifying notion that recovers each of the above notions as special cases. (Joint work with Sean Sather-Wagstaff)
Speaker: Jon Totushek  
Title: Homological dimensions with a semidualizing module C (I,II)  
Dates: April 5, 19

Abstract: Let R be a local noetherian ring with maximal ideal \( m \) and residue field k. Auslander, Buchsbaum, and Serre proved that R is a regular local ring if and only if \( pd_R(M) \) for every R-module M. Similar results characterize complete intersection rings with the complete intersection projective dimension \( CI - pd_R(M) \). This talk will explore other types of homological dimensions and what types of information we can get from them.

Speaker: Sean Sather-Wagstaff  
Title: The shape of some free resolutions  
Date: February 22

Abstract: Every module M over a commutative ring R is a homomorphic image of a free R-module. Iterating this procedure, one sees that M has a free resolution, that is, there is an exact sequence,

\[
\cdots \to F_2 \to F_1 \to F_0 \to M \to 0
\]

such that each \( F_i \) is free. This is the starting point, in some sense, for homological algebra. One goal in this area is to give explicit algorithms for describing such resolutions. In general, this problem is intractable. In this talk, after providing some some background information, I will describe some situations where this problem can be solved.
6 Fall 2012

Speaker: Aaron Feickert
Title: Move trees and the minimax algorithm
Date: November 30

Abstract: How would you teach a computer to play games against a human player? Using move trees and the minimax algorithm, we can develop techniques for simple game strategies. We will discuss how the algorithm works and use it to play simple games. We will also discuss evaluation functions and ways they apply to more complicated games like chess.

Speaker: Artem Novozhilov
Title: On mathematical modeling of biological evolution
Date: November 2

Abstract: In my talk I will give an introduction to the mathematical approaches to model biological evolution. In particular, I will formulate the celebrated Eigen’s quasispecies model and state the basic results concerning the quasispecies concept and the error threshold. Connections with the statistical physics will be presented. I will conclude my presentation with possible research directions.

Speaker: Brandon Goodell
Title: Factorization and Sequences of order preserving homomorphisms II
Date: October 5

Speaker: Sean Sather-Wagstaff
Title: The yoga of chain complexes
Date: September 21

Abstract: A large part of commutative algebra research is focused on the study of modules over commutative rings. However, some results about modules can only be proved by considering a larger class of objects: chain complexes. I will discuss some aspects of this area, giving lots of background. I will only assume familiarity with ideas from Math 721.
Speaker: Maria Alfonseca  
Title: Reconstructing a body in $R^n$ from lower dimensional and information about it  

Abstract: A recurring question in convex analysis is the determination of a convex body $K$ from a given set of lower dimensional information about it, such as the volume of its sections or its projections. In this talk we will review several problems about the determination of $K$ from its hyperplane sections. Different problems arise depending on the type of sections considered (central sections, maximal sections, sections tangent to a prescribed ball inside of $K$, etc.) In all cases, the main questions are the following,

- If all the sections of $K$ have the same volume, is $K$ a ball?
- If all the sections of $K$ have the same volume as the corresponding sections of $L$, is $K=L$?

And the answers may be very different (and surprising) for each type of problem.

Speaker: Brandon Goodell  
Title: Factorization and sequences of order-preserving homomorphisms  
Date: September 7  

Abstract: Factorization usually investigates how to break an element in an integral domain into a product of irreducibles. This usually isn’t possible in the exotic world of purgatory and antimatter domains. In this talk we will look into some natural ways to peel the layers of the factorization onion by examining the group of divisibility of an integral domain, and we will develop some structure theorems about groups of divisibility.