Partial orders

Worksheet 13

A partial ordering on set X is a relation which is reflexive, transitive, and antisymmetric.

Show that the following relations define partial orderings on the underlying sets.

1. $\{(n,m) : n, m \in \mathbb{Z}$ such that $n - m \geq 0\}$.
2. $\{((a,b),(c,d)) : (a,b),(c,d) \in \mathbb{Z} \times \mathbb{Z}$ such that either $a < c$ or $a = c$ and $b \leq d\}$.
3. $\{((a,b),(c,d)) : (a,b),(c,d) \in \mathbb{Z}\times\mathbb{Z}$ such that either $b < d$ or $b = d$ and $a \leq c\}$.
4. If S is a set then $\{(A,B) : A, B \in P(S)$ with $A \subseteq B\}$.

A partial ordering P on X is a total ordering if given any $x, y \in X$ either $(x,y) \in P$ or $(y,x) \in P$.

Which of the preceding partial orderings are total orderings?