1. Let \( A = \begin{bmatrix} x + 2 & 2 & -4 & 6 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & x + 3 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \) and \( B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2x & -5 \\ 0 & 1 & 2 \end{bmatrix} \). Find all values of \( x \) such that \( \det(A) = \det(B) \).

2. Let \( A \) and \( B \) be \( 4 \times 4 \) matrices such that \( \det(A) = -2 \) and \( B \) is produced by performing the following sequence of operations on \( A \):

\[
R_1 \leftrightarrow R_2, 4R_1 \rightarrow R_1, 4R_2 + R_3 \rightarrow R_3, \sqrt{2}R_1 \rightarrow R_1, 2R_3 + R_1 \rightarrow R_1, \frac{1}{2}R_4 \rightarrow R_4
\]

a) Find \( \det(B) \).

b) Find \( \det(4(A^{-1})^2B^T) \).

c) Find \( \det(BA - 3B^2AI_3(BA)^{-1}A) \).

3. Let \( A \) be an \( n \times n \) matrix and let \( H = \{ B \in \mathbb{M}_{n \times n} : AB = BA \} \). Determine if \( H \) is a subspace of \( \mathbb{M}_{n \times n} \).

4. Consider the transformation \( F : \mathbb{P}_3 \rightarrow \mathbb{R}^2 \) given by \( F(p(x)) = \begin{bmatrix} p(0) \\ p(0) \end{bmatrix} \).

   (a) Show that \( F \) is a linear transformation.

   (b) Describe the kernel of \( F \).

   (c) Does \( S = \{ x^2, x^3 - 2x, 3x^3 + \pi x^2 + 5x \} \) span the kernel of \( F \)?

5. (a) Define a transformation \( T : \mathbb{M}_{2 \times 2} \rightarrow \mathbb{R} \) by \( T(A) = \det(A) \). Determine if \( T \) is a linear transformation. Clearly justify your conclusion.

   (b) Let \( H = \left\{ \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} : a = -2c, f = 2e + d \right\} \). Is \( H \) a subspace of \( \mathbb{M}_{2 \times 3} \).

6. (Bonus) Let \( T : V \rightarrow W \) be a linear transformation. Prove that \( T \) is one-to-one if and only if \( \text{Ker}(T) = \{0\} \). (Recall a transformation \( T : V \rightarrow W \) is said to be one-to-one if for all \( v_1, v_2 \) in \( V \), \( T(v_1) = T(v_2) \) implies \( v_1 = v_2 \).)