Using Bézier Curves to Design Self-Adapting Conformal Phased-Array Antennas

Sayan Roy, *Student Member, IEEE*, Bilal Ijaz and Benjamin D. Braaten, *Member, IEEE*
Department of Electrical and Computer Engineering
North Dakota State University
Fargo, ND, USA 58108-6050
Email: sayan.roy@my.ndsu.edu, benbraaten@ieee.org

***Abstract*—A new technique for computing the location of each antenna element in a conformal array using general Bézier curves is presented here. More specifically, two cubic polynomial Bézier curves are used to compute the location of each antenna element in a conformal 1×4 phased-array antenna on a wedge- and cylindrical-shaped surface for various bend-angles and radius of curvature values, respectively. The accuracy of these location values are validated by comparison with analytical computations and published literature.**

***Index Terms*—Bézier Curves, Conformal Antennas, Phased-arrays.**

I. INTRODUCTION

Lately, conformal antennas for space-born applications, wearable networks, search and rescue personnel, and military systems [1] have gained popularity because new material and electronics manufacturing techniques [2] have been recently developed to improve the capabilities. However, in some instances, the surface that a conformal antenna may be attached to can change shape and this can have a negative impact on the radiation properties of the conformal antenna [3]. Various mechanical and electrical compensation techniques have been developed to recover the radiation pattern of a conformal antenna on a changing surface [1] and a key component of each of these compensation methods is to know the location of each antenna element on the conformal surface.

The objective of this paper is to present a new method for computing the location of each antenna element in a conformal array on a wedge- and cylindrical-shaped singly-curved surface using third-order Bézier curves [4]. An example of a Bézier curve is shown in Fig. 1. Then, the accuracy of the Bézier curves is shown by modeling problems with known surfaces and determining the pattern of the 1×4 conformal array on the cylindrical-shaped surface in Fig. 2, and comparing the results to the values reported in [1].

II. DEFINING ARBITRARY SURFACES USING BÉZIER CURVES

A third-order Bézier curve (shown in Fig. 1) consists of four control points, denoted as $B1$, $B2$, $B3$, and $B4$, and four weights [4]. For simplicity, only the rational Bézier curve [4] of order three will be considered in this paper and symmetry about the z-axis will be assumed. Then, the following equations can be evaluated to determine the values of $B1$, $B2$, $B3$, and $B4$ [4]:

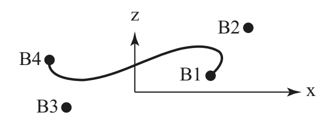

Fig. 1. A third-order rational Bézier Curve with four control points on x-z plane

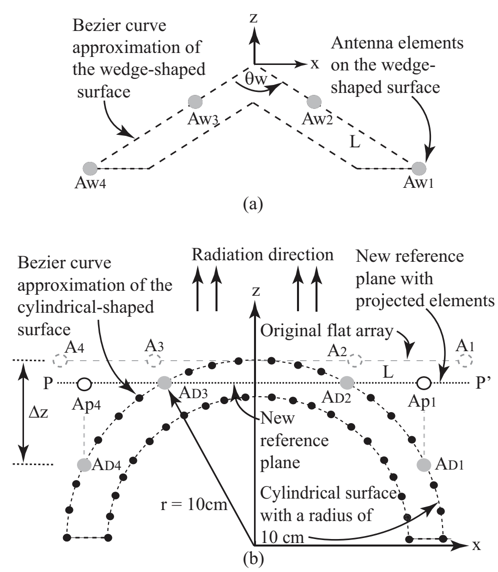

Fig. 2. Four element antenna array on (a) wedge- and (b) cylindrical-shaped surfaces.

$$x_B(t) = (1-t^3)x_{B1} + 3t(1-t)^2 x_{B2} + 3t^2(1-t)x_{B3} + x_{B4} \quad (1)$$

$$z_B(t) = (1-t^3)z_{B1} + 3t(1-t)^2 z_{B2} + 3t^2(1-t)z_{B3} + z_{B4} \quad (2)$$

where $(x_{BN},\ z_{BN})$ denote the coordinates of control point BN and N = 1, 2, 3 and 4. The values computed by (1) and (2) are the $x-$ and $z-$values, respectively, of the Bézier curve. Then, by knowing the inter-element spacing, geometry and operating frequency of the antenna design, the location of each antenna element can be computed using (1) and (2).

TABLE I
COORDINATES OF THE ARRAY ON THE WEDGE-SHAPED SURFACE.

Coordinates	Analytical (mm)	Bézier Curve (mm)
(x_{W1}, z_{W1})	(67.25,-67.25)	(67.25,-67.25)
(x_{W2}, z_{W2})	(22.52,-22.52)	(22.52,-22.52)
(x_{W3}, z_{W3})	(-22.52,-22.52)	(-22.52,-22.52)
(x_{W4}, z_{W4})	(-67.25,-67.25)	(-67.25,-67.25)

TABLE II
COORDINATES OF THE ARRAY ON THE CYLINDRICAL-SHAPED SURFACE.

Coordinates	Analytical (mm)	Bézier Curve (mm)
(x_{D1}, z_{D1})	(79.0, -38.69)	(79.0, -43.14)
(x_{D2}, z_{D2})	(29.89, -4.57)	(29.89, -5.45)
(x_{D3}, z_{D3})	(-29.89, -4.57)	(-29.89, -5.45)
(x_{D4}, z_{D4})	(-79.0, -38.69)	(-79.0, -43.14)

III. VALIDATION OF THE PROPOSED METHOD

A. The Wedge-shaped Surface

First, the location of the elements in the 1×4 array on the wedge-shaped surface in Fig. 2(a) was computed using the Bézier curves. The inter-element spacing of the array was assumed to be λ/2, the operating frequency was set to 2.47 GHz and the wedge angle was defined to be θ_w = 90°. Each side of the wedge was first considered as a straight line and described using:

$$x(t) = (1-t)x_1 + tx_2 \quad (3)$$

$$y(t) = (1-t)y_1 + ty_2. \quad (4)$$

Then, the terms in (3) and (4) were equated to the terms in (1) and (2) with the same order to analytically solve for the location of control points of the Bézier curve. The control points were determined to be the following (in mm): B1 = (0,0), B2 = (-44.8,-44.8), B3 = (22.4,-22.4) and B4 = (67.2,-67.2) and the location of each antenna element computed using the Bézier curves is shown in column 3 of Table I. The analytical values using trigonometry were also computed and are reported in column 2 of Table I to show agreement.

B. The Cylindrical-shaped Surface

Next, the location of the elements in the array on the cylindrical-shaped surface shown in Fig. 2(b) were computed using the Bézier curves. In this case, the radius of the cylindrical surface was assumed to be r = 10.0 cm, the inter-element spacing was $\lambda/2$ and the operating frequency was set to 2.47 GHz. Because of the cylindrical nature of the surface, a cubic GC^2 Bézier curve representation of the surface was required [4]-[5] and will include some error. Then, using (1) and (2) the control points were determined to be (in mm): B1 = (0,0), B2 = (55.2,0), B3 = (100,-55.2) and B4 = (100,-100), and the computed location of each antenna element is shown column 3 of Table II. Again, for comparison, the location of each antenna element was computed using trigonometry and is shown in column 2 of Table II for comparison. Also, the shape of the cylindrical surface was computed analytically and using the Bézier curves. These results are shown in Fig. 3 and it is shown that the Bézier curves can be used to accurately model

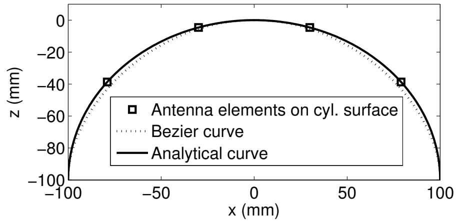

Fig. 3. Comparison between the cylindrical surface computed using analytical methods and the Bézier curves.

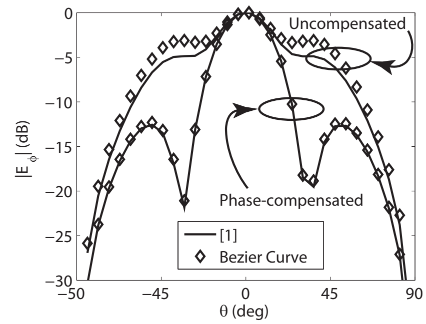

Fig. 4. Normalized uncompensated and phase-compensated patterns.

the cylindrical surface. Finally, the phase-compensated [1] pattern of the 1×4 array on the cylindrical surface described by the Bézier curves was computed and compared to the results reported in [1]. A good comparison between these results is shown in Fig. 4; indicating that the Bézier curves presented here can be used to accurately design a self-adapting conformal array that uses the phase-compensation technique described in [1].

IV. CONCLUSION

A new technique for determining the shape of a wedge- and cylindrical-shaped surface was presented here. In particular, general Bézier curves were used to compute the location of the antenna elements on these surfaces and then this information was used to implement a phase-compensation technique for radiation pattern recovery. Finally, computations were validated with a comparison to analytical and published results.

REFERENCES

[1] B. D. Braaten, S. Roy, S. Nariyal, M. A. Aziz, N. F. Chamberlain, I. Irfanullah, M. T. Reich, and D. E. Anagnostou, "A Self-Adapting Flexible (SELFLEX) Antenna Array for Changing Conformal Surface Applications," *IEEE Trans. on Antennas and Propagation*, pp. 655 - 665, vol. 61 no. 2, 2013.

[2] V. Marinov, O. Swenson, R. Miller, F. Sarwar, Y. Atanasov, M. Semler and S. Datta, "Laser-enabled advanced packaging of ultrathin bare dice in flexible substrates," *IEEE Trans. Comp., Packaging and Manu. Tech.*, vol. 2, no. 4, pp. 569 - 577, Apr. 2012.

[3] R. L. Haupt *Antenna Arrays: A Computational Approach*, Hoboken, NJ: Wiley, 2010.

[4] L. Piegl and W. Tiller, *The NURBS Book, 2nd ed.*, Springer-Verlag, NewYork, 1996.

[5] A. Riškus, "Approximation of a cubic Bézier curve by circular arcs and vice versa," *Info. Techno. and Control,* vol. 35, no. 4, pp. 371-378, 2006.