On the Spectral Domain Moment Method Solution of Electric Field Integral Equations for the Analysis of Printed Dipoles in Anisotropic Layers

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***Abstract*—Simplified electric field integral equations written in a manner suitable for numerical evaluation with the spectral domain moment method are presented here. In particular, these integral equations are written in terms of spectral domain immittance functions for multi-conductor problems and it will be shown that these expressions have an additional benefit, in terms of simplicity, over other methods because a numerical derivative is not required. The numerical computation of these expressions are validated by comparison with published results and commercial software.**

I. INTRODUCTION

Solving for the electromagnetic characteristics of printed antennas in complex environments has been of great interest to antenna designers for many years. This is because modern communication systems are being required to operate with surroundings that include multiple scattering surfaces, various dielectric materials and other antennas. Printed dipole antennas are among the various antennas used in a wireless system and in many cases, these antennas are printed on a multi-layer substrate with other circuitry and conducting interconnects. Because of the narrow band nature of a printed dipole, it is important to be able to accurately model the multi-layered substrate especially because it has been shown that these substrates have uniaxial anisotropic properties [1]-[2].

The interaction between printed dipoles and the surrounding environment in layered dielectrics has been studied and was reported in [3]-[4]. This work was then extended to include the mutual coupling between printed dipoles in layered uniaxial anisotropic dielectrics in [2]. Throughout much of this work Hertzian vector potentials and the spectral domain immittance functions [1] were used to numerically evaluate the characteristics of the printed dipoles because these functions do not require a surface derivative.

The objective of this paper is to present new electric field integral equations suitable for numerical evaluation with the spectral domain moment method [5]. Furthermore, these electric field integral equations will be written in terms of the multi-conductor spectral domain immittance functions reported in [2] and do not require a surface derivative.

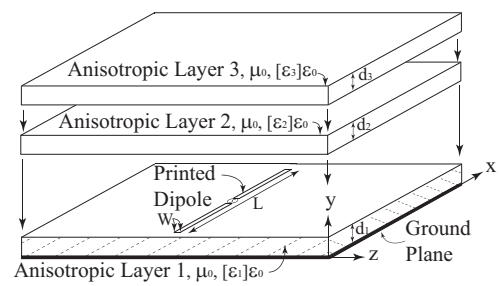

Fig. 1. Expanded view of the printed dipole in 3 layers of anisotropic material.

II. THE NEW ELECTRIC FIELD INTEGRAL EQUATIONS FOR SOLUTION BY THE SPECTRAL DOMAIN MOMENT METHOD

For illustration, consider the multi-layer problem defined in Fig. 1. The problem consists of a printed dipole embedded in three layers of uniaxial anisotropic dielectric substrates. The thickness of each dielectric layer is denoted as d_k and the permittivity is denoted as $[\varepsilon_k]$ for k = 1, 2 and 3. The medium above layer 3 is air and it is assumed that each dielectric substrate is infinite in length in the x-z plane. The bottom layer is grounded.

As shown in [2], the spectral domain immittance functions $\widetilde{Z}_{xx}$, $\widetilde{Z}_{xz}$, $\widetilde{Z}_{zx}$ and $\widetilde{Z}_{zz}$ represent the Green's functions in the spectral domain. Therefore, the Green's functions in the spatial domain, denoted as Z_{xx}, Z_{xz}, Z_{zx} and Z_{zz}, are the inverse Fourier transforms of $\widetilde{Z}_{xx}$, $\widetilde{Z}_{xz}$, $\widetilde{Z}_{zx}$ and $\widetilde{Z}_{zz}$, respectively. Using these notations, the x-component of the electric field in any region can be written as [6]:

$$E_x(x,y,z) = \frac{1}{(2\pi)^2}\int_{z'}\int_{x'} \Bigg[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \widetilde{Z}_{xx}(\alpha,\beta)e^{jR}d\alpha d\beta J_x(x',z') + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \widetilde{Z}_{xz}(\alpha,\beta)e^{jR}d\alpha d\beta J_z(x',z')\Bigg]dx'dz' \quad (1)$$

where $R = \alpha(x - x') + \beta(z - z')$ and the prime notation indicates the location of the surface currents. Next, the current is defined in terms of the following basis functions:

$$J_x(x', z') = \sum_{n=1}^{N} I_{xn} r_{xn}(x', z') \tag{2}$$

and

$$J_z(x', z') = \sum_{n=1}^{N} I_{zn} r_{zn}(x', z') \tag{3}$$

where I_{xn} and I_{zn} are the unknown magnitudes of the x- and z-components of the surface current, respectively. The known expansion functions are $r_{xn}(x', z')$ and $r_{zn}(x', z')$. Since Galerkin's method is being used, the weighting functions $w_{xm}(x, z)$ and $w_{zm}(x, z)$ will be defined to be the same as the expansion functions $r_{xn}(x', z')$ and $r_{zn}(x', z')$, respectively. The weighting functions will be indexed by the variable m and are defined at x and z. Substituting (2) and (3) into (1), taking the inner product $\langle w_{xm}(x, z), E_x(x, y, z)\rangle$ and factoring results in the following expression:

$$\begin{aligned}(2\pi)^2 \langle w_{xm}(x, z), E_x(x, y, z)\rangle = \\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\widetilde{w}_{xm}(-\alpha, -\beta)\widetilde{Z}_{xx}(\alpha, \beta)\widetilde{r}_{xn}(\alpha, \beta)\right] d\alpha d\beta \\ + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\widetilde{w}_{xm}(-\alpha, -\beta)\widetilde{Z}_{xz}(\alpha, \beta)\widetilde{r}_{zn}(\alpha, \beta)\right] d\alpha d\beta\end{aligned} \tag{4}$$

where

$$\widetilde{r}_{x(z)n}(\alpha, \beta) = \int_{z'}\int_{x'} r_{x(z)n}(x', z') e^{-j\alpha x'} e^{-j\beta z'} dx' dz'. \tag{5}$$

Equation (5) is the Fourier transform of the basis functions r_{xn} and r_{zn} in (2) and (3), and is denoted as $\widetilde{r}_{xn}$ and $\widetilde{r}_{zn}$, respectively. Therefore, if the basis functions are chosen appropriately, an analytical expression for (5) could be derived and substituted directly back into the desired expressions.

The steps leading to (4) are key to implementing the new electric field integral equations in electromagnetics problems. Notice that the integration of (4) is entirely in the spectral domain and the electric field values are in the spatial domain. Also notice that the expressions in (4) do not contain a numerical derivative and are now more suitable for implementing in the spectral domain moment method [5]. Finally, it should be noted that a similar expression to (4) for the z-component of the electric field can be derived and the complete expressions for $\widetilde{Z}_{xx}$, $\widetilde{Z}_{xz}$, $\widetilde{Z}_{zx}$ and $\widetilde{Z}_{zz}$ are reported in [6].

III. NUMERICAL RESULTS

For validation, the dipole problem in Fig. 1 was simplified to a single anisotropic problem and evaluated for d_1 = 1.58 mm and $d_2 = d_3$ = 0. The length of the dipole was defined to be L = 15.0 cm and the width was defined to be W = 0.5 mm. Since the surfaces in this work are planar, a two-dimensional piecewise sinusoidal (PWS) basis function was chosen. It was assumed that the current on the dipole in Fig. 1 did not vary with respect to the width of the conductor and only varied with respect to the length. Therefore thin-wire assumptions were enforced. The results from the computations of (4) are sucessfully compared to the results from the commercial software ADS [7] in Fig. 2 for various values of n, where $n = \sqrt{\varepsilon_x/\varepsilon_y}$ is the anisotropy ratio. Further validation of multiple printed dipoles in two- and three-uniaxial anisotropic layers is reported in [2] and [6].

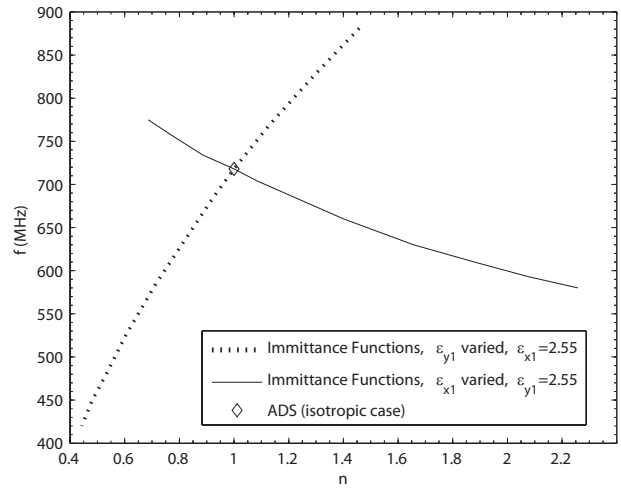

Fig. 2. Resonant frequency of a printed dipole on a single anisotropic substrate for various values of n.

IV. CONCLUSION

Simplified electric field integral equations written in terms of spectral domain immittance functions for numerical evaluation with the spectral domain moment method were derived and presented. It was shown that a numerical derivative is not required and that the integration of the immittance functions are completely in the spectral domain. Finally, a numerical example of a single printed dipole on an anisotropic substrate was presented for validation.

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