On the Numerical Integration of Spectral Domain Immittance Functions for Multiple Printed Dipoles in Layered Uniaxial Anisotropic Dielectrics

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***Abstract*—Spectral domain immittance functions can be used to numerically solve for the unknown surface currents on printed dipoles in layered anisotropic dielectrics. This solution requires the use of the spectral domain moment method and numerical integration. This paper presents a hybrid polar- and rectangular-numerical integration technique on the α-β plane useful for efficient evaluation of the spectral domain moment method and is defined in a manner that avoids unwanted poles. For validation, the convergence of the numerically computed input impedance of a printed dipole is presented.**

***Index Terms*—Spectral domain immittance functions, printed dipoles**

I. INTRODUCTION

Printed dipoles on a grounded anisotripic dielectric substrate have a narrow bandwidth (BW) because of the close proximity of the dipoles to the ground plane and the resonant nature of the dipole arms [1]. Because of this narrow BW, it is important to be able to accurately model the printed dipoles in a complex radiating environment. One method to numerically model a printed dipole accurately is to use the spectral domain immittance functions reported in [2]-[4]. It has been shown that these functions can be solved using the spectral domain moment method (SDMM) reported in [5]. Associated with this moment method is an inner product and a numerical integration of the spectral domain immittance functions. This numerical integration is further complicated by the location of poles which result in unwanted divergence of the functions. Therefore, the objective of this paper is to present a method of dividing up the transform plane (α-β plane) on which the spectral domain immittance functions are defined that avoids the locations of the unwanted poles. This numerical integration will be helpful for designers and researchers using this method to model printed antennas.

II. FUNCTIONS FOR THE SDMM

For this paper, the problem in Fig. 1 will be considered. In particular, two printed dipoles are defined between uniaxial anisotropic dielectric layers. Dipole 1 is defined on layer 1 and dipole 2 is defined on layer 2. Next, expressions for the electric

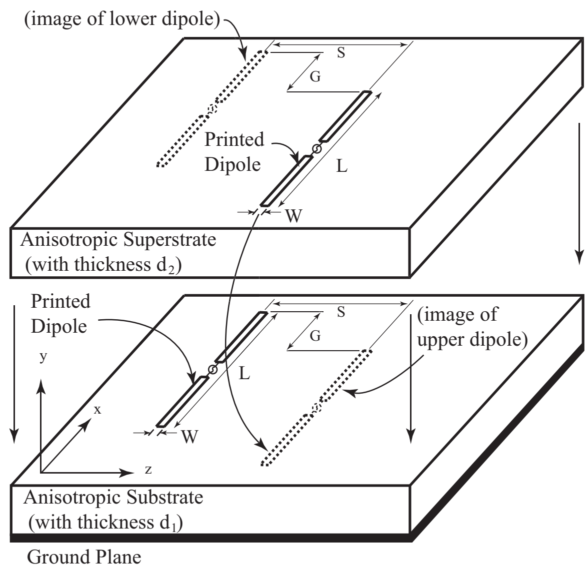

Fig. 1. Illustration of two printed dipoles in two layers of uniaxial anisotropic dielectrics.

field in each layer are written in terms of Hertzian vector potentials [4] and used to enforce the boundary conditions between the anisotropic layers. Then, after several algebraic steps, the following expressions can be used in the SDMM to numerically solve for the unknown currents on the printed dipoles in Fig. 1 [4]:

$$(2\pi)^2\langle w_{xm}(x,z), E_x(x,y,z)\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\widetilde{w}_{xm}(-\alpha,-\beta)\widetilde{Z}_{xx}(\alpha,\beta)\widetilde{r}_{xn}(\alpha,\beta)\right]d\alpha d\beta + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\widetilde{w}_{xm}(-\alpha,-\beta)\widetilde{Z}_{xz}(\alpha,\beta)\widetilde{r}_{zn}(\alpha,\beta)\right]d\alpha d\beta \quad (1)$$

where $\widetilde{r}_{x(z)n}$ is the Fourier transform of the known basis function $r_{x(z)n}$, $\widetilde{w}_{xm}$ is the Fourier transform of the known weighting function w_{xm}, $\langle\ \rangle$ denotes the inner product, E_x is

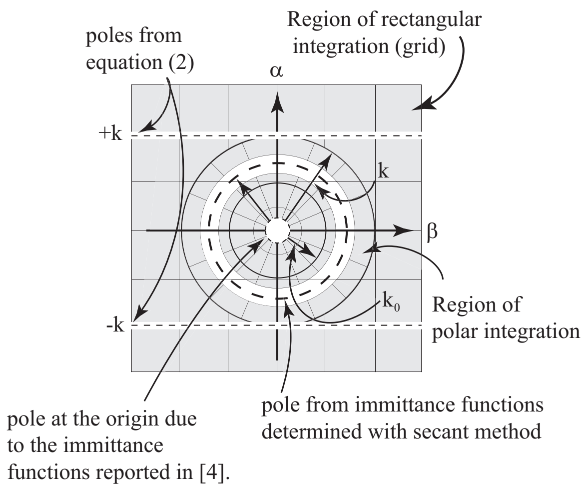

Fig. 2. An illustration of the hybrid polar- and rectangular-integration on the α-β plane.

the x-component of the electric field and $\widetilde{Z}$ denotes the spectral domain immittance function in the transform domain of α and β. Next, the piecewise sinusoidal functions reported in [4] were defined for both the basis and weighting functions (i.e., Galerkin's method). It can be shown that the two-dimensional Fourier Tranform of the basis function for the i^{th} segment along the dipole can be written as:

$$\widetilde{r}_i(\alpha,\beta) = \frac{2}{\sin(k\Delta x)}\left(\frac{1}{k+\alpha} + \frac{1}{k-\alpha}\right)e^{-j\alpha x_i}F_1F_2 \quad (2)$$

where

$$F_1 = \sin\frac{\Delta x(k+\alpha)}{2}\sin\frac{\Delta x(k-\alpha)}{2} \quad (3)$$

and

$$F_2 = e^{-j\beta z_i}\frac{W\sin(\beta W/2)}{\beta W/2}e^{-j\beta W/2}. \quad (4)$$

Also, $\Delta x = L/N$ is the segment size along the x-direction of the dipole for the moment method evaluation, W is the width of the printed dipole and N is the number of segments. Equation (2) can now be substituted directly into (1).

III. Numerical Integration

The location of the poles in (1) has led to the use of polar integration from the origin to a circle of radius k and rectangular integration on the remainder of the α-β plane. Both types of numerical integration are shown in Fig. 2. The first pole is in the denominator of (1) and has the form $\gamma_{e1} + \varepsilon_{12}\gamma_0\coth(\gamma_{e1}d_1)$. The secant method is used to find the root to this equation, and it is located between k_0 and k (as shown in Fig. 2). Similarly, it can be seen in (2) that a pole exists at $\pm k$. This is also shown in Fig. 2. In addition, the immittance functions in (1) each have a pole at the origin (as shown in [4]), which is illustrated in Fig. 2. The shaded areas in Fig. 2 indicate the region of the plane in which the numerical integration is performed. The regions around the poles are removed from the numerical integration to avoid the singularity associated with the expressions; as indicated by the white areas with the poles shown by dotted lines.

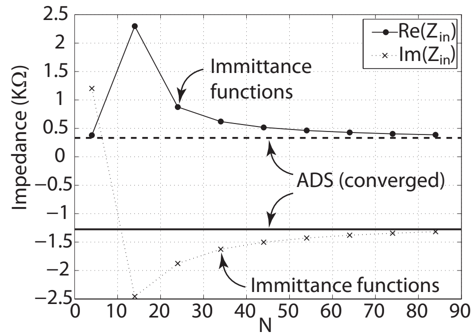

Fig. 3. Input impedance convergence for the $0.7\lambda_0$ dipole.

IV. Validation

For validation of the integration defined on the α-β plane in Fig. 2, the input impedance of a printed dipole was computed for N = 4, 14, 24, ...84. The length of the dipole was L = $0.7\lambda_0$, the width was W = 4a (a = $0.0001\lambda_0$), and the substrate had a thickness of d = $0.1016\lambda_0$ and a permittivity of ε_r = 3.25. The results from these computations are shown in Fig. 3. To test the accuracy of this method, the values of input impedance computed using the hybrid integration were compared to those computed with the commercial software ADS [6]. Excellent agreement was obtained, as illustrated in Fig. 3. Further validation of microstrip patches and printed dipoles on anisotropic substrates is reported in [3] and [4].

V. Conclusion

A numerical integration technique on the α-β plane for the spectral domain moment method has been presented in this paper. This technique consisted of a hybrid polar- and rectangular-grid that avoided the unwanted poles in the plane. For validation, a printed dipole on an isotropic substrate was evaluated and successfully compared to the results of a commercial software package and published literature.

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