Circuit Analysis

- Foundation includes charges -
  - Charge is bipolar - meaning we have both pos & neg. charges
  - Unit of charge is a Coulomb
  - There are 6.24x10^18 charges in a single Coulomb.

- There are two orientations of charges we consider:
  - Separation & movement
  - 

- Electrical force creates electrical field between charges - known between points - known in a cell as a voltage as an electric current or electrical potential, or simply current.

Voltage - Defined as the energy per unit charge created by a separation or

\[
V = \frac{dW}{dq} \quad \text{(V or (J/C)}
\]

Where \( V \) = voltage in volts (V)
\( W \) = energy in joules (J)
\( q \) = charge in coulombs
\textbf{Current} - Defined as Coulombs per unit time.

\[ i = \frac{dq}{dt} \left( \frac{C}{s} \right) \]

where \( i \) = current in amperes,
\( q \) = charge in coulombs,
\( t \) = time in seconds.

\textbf{Passive Sign Convention}

When the voltage drop is in the same direction as the current, then we use a positive sign in any expression that relates \( V \) to \( i \). Otherwise, we use a negative sign.

\textbf{Power and Energy}

Power is defined as the energy expended (or absorbed) per unit time.

\[ P = \frac{dW}{dt} \Rightarrow 1W = 1 \frac{J}{s}. \]

where \( P \) = power in Watts (W).

\[ P = \frac{dW}{dt} = \frac{dW}{dq} \cdot \frac{dq}{dt} \]

\[ \Rightarrow P = Vi \quad (W) \]

\textbf{Power equation.}
Now we use the passive sign convention for power computations.

**Ex. 1:**

\[ P = VI \] relates voltage to current, \( \Rightarrow P = -VI \)

\[ = -8 \text{W}. \]

\( \Rightarrow \) elt. is supplying power.

**Ex. 2:**

\[ P = VI \]

\[ = -(2 \cdot 2) \]

\[ = 4 \text{W} \Rightarrow \text{absorb power.} \]

Since \( \Delta I \) is given into the pos. term. of an elt., the elt. is absorbing \( 4 \text{W} \) of power.
Ex. 3 Power absorbed or supplied by e.m.

\[ + \quad 12V \quad + \quad \text{supply} \quad P = 96W \]
\[ - \quad 12V \quad - \quad \text{or absorbed} \quad = 96W \text{ of pur.} \]

Ex. 4 Power absorbed or supplied by e.m.

\[ + \quad 2A \quad + \quad \text{Power absorbed is 36W or} \]
\[ - \quad 4A \quad - \quad 8W \text{ supplied.} \]

Circuit Element Models

2 Model types - Passive elements - store/use pur.
- Active elements - supplies pur.

Model - Basic Circuit Elements

i) Two terminals

ii) Cannot be divided to anything smaller

Active (Sources)
Passive Elements

$V_s = \mu V_i$ 

- Dependent Voltage Source

$\mathbf{\beta} \mathbf{\eta}$

- Dependent Current Source

$V \leq R \Rightarrow$ Resistor, units Ohms, ($\Omega$).

$- \mathbf{\beta} \mathbf{\eta}$

- Denoted: Resistance

$\mathbf{\chi}$

- $C \Rightarrow$ Capacitor, units Farads, ($F$)

- Denoted: Capacitance

$L \Rightarrow$ Inductor, units Henrys, ($H$).

- Denoted: Inductance

Conductance is defined as $G = \frac{1}{R}$ (Siemens) or ($S$).

Ohm's Law

Consider: $V \leq R$

$[V = \mathbf{I}R]$

- Note in this definition it is entirely the $+V$ term.
Ex1:
Consider
Find $P_R$

\[
\begin{array}{c}
\text{10A} \\
\uparrow \\
\text{V} \geq 10.5 \text{V}
\end{array}
\]

$P_R = VI$. To find $V$ we use $V = iR$ - Ohm's law.

$\Rightarrow V = 10A \cdot 10.5 \text{V} = 100 \text{V} \Rightarrow P = 100 \text{V} \cdot 10 \text{A} = \boxed{1000 \text{W}} = \boxed{1 \text{ KW}}$

Power absorbed by $R$ or supplied by the source.

Ex2: Consider:

\[
\begin{array}{c}
\text{2V} \\
\uparrow \\
\text{V} = 2 \text{V}
\end{array}
\]

Find $P_R$. $V = 2 \text{V}$

From Ohm's law, $V = iR \Rightarrow i = \frac{2}{10} = 0.2 \text{A} = 200 \text{mA}$

$\Rightarrow P_R = 2 \text{V} \cdot 0.2 \text{A} = \boxed{0.4 \text{W}} = \boxed{0.4 \text{KW}}$

**Kirchhoff's Laws**

A circuit is said to be solved if every voltage across and the current through every element is known.

A node is defined to be the point where two or more circuit elements meet.
Kirchhoff's Current Law: The algebraic sum of all the currents at any node equal to zero, or
\[ \sum_{m=1}^{M} (i_{m})_{in} = \sum_{n=1}^{N} (i_{n})_{out}. \]
Short hand - **KCL**

Kirchhoff's Voltage Law: The algebraic sum around any closed path is equal to zero, or
\[ \sum V = 0. \]
Short hand - **KVL**

Example:

\[ V - 10V - SV + V + 3V = 0 \]
\[ V = 5V \]

Write the sign of the voltage to be the first sign we see around the loop.
Ex:

\[
\begin{align*}
I_2? & \quad M \quad 5A \\
\end{align*}
\]

\[\rightarrow A_{mp} \text{ KCL at node } M,\]
\[\Rightarrow 10A - 5A - I = 0,\]
\[\Rightarrow I = 5A\]

Define currents entering a node to be pos. & leaving to be neg.
\[I_2 \quad V^+ \quad V^-\]

Ex:

\[
\begin{align*}
10V & \quad +2V \quad +1V \\
L_1 & \quad V \quad \quad L_2 \\
-10V & \quad 8V
\end{align*}
\]

1st way

\[
I = 2A - 1A = 1A
\]
\[
V = 10V - 2V = 8V
\]

Next, \[L_3 \Rightarrow -8V - V + V' = 0\]
\[\Rightarrow -8V - 10V = -V'\]
\[\Rightarrow V' = 18V\]

2nd way

A round \[L_1: -12V + 2V + V = 0\]
\[\Rightarrow V = 10V\]

A round \[L_2: -8V - V + V' = 0\]
\[\Rightarrow 8V + 10V = V'\]
\[\Rightarrow V' = 18V\]
Ex:

\[
\begin{align*}
\begin{array}{c}
\text{Ex}:
\end{array}
\end{align*}
\]

Determine \( i_s + i_d \) if \( V_i = 2.5V \)

KVL around \( L_1 \):
\[-10V + V_i + V_3 + V_5 = 0 \]
\[\Rightarrow V_i + V_3 + V_5 = 10V \]

From observation \( V_1 = V_3 \) \( \Rightarrow 2V_1 + V_3 = 10V \)
\[\Rightarrow 5V + V_3 = 10 \]
\[\Rightarrow V_3 = 5V. \]

\[\Rightarrow i_1 = \frac{V_3}{20} = \left[ \frac{1}{4} A \right] \text{ or } \left[ 250mA \right] \]

We also know \( N_1 \), \( i_s = i_1 + i_d \)

Need \( i_d \). We know \( V_3 = 5V \), then around \( L_2 \) we have:

\[V_4 + V_5 = V_3.\] Since the loop (es. shunt \( i_d \), \( V_4 \)) \( V_4 = V_5 \)
\[\Rightarrow 2V_4 = V_3 \Rightarrow V_4 = \frac{V_3}{2} = 2.5V \Rightarrow i_d = \frac{2.5V}{10} = .25mA \]

\[ \therefore \] \( i_s = i_1 + i_d \)
\[= 250mA + .25mA \]
\[= 250mA \]


Resistors in Series

Consider the following circuit:

\[ V_s \quad \text{series connected circuit} \]

where \( R_1 \neq R_2 \neq R_3 \neq R_4 \neq R_5 \neq R_6 \neq R_7 \)

Applying KVL gives:

\[ V_s = V_s + I_s R_1 + I_s R_2 + \ldots + I_s R_7 \]

\[ \Rightarrow V_s = I_s (R_1 + R_2 + \ldots + R_7) \]

\[ \Rightarrow \text{Req.} \]

\[ \Rightarrow \text{we define the equivalent resistance of the series combination as: } \text{Req.} = R_1 + R_2 + \ldots + R_7. \]

\[ \Rightarrow \text{in general: } \text{Req.} = \sum_{i=1}^{N} R_i \]

\[ \Rightarrow \text{Trade-off is info. is lost about the circuit, such as individual parts.} \]

\[ \Rightarrow \text{The above circuit can be written as: } \]

\[ I_s \quad a \quad \text{Req.} \]

\[ V_s \quad \text{b} \]
Resistors in Parallel

Consider the following circuit:

\[ \begin{align*}
I_5 &= I_1 + I_2 + I_3 + I_4 \\
(\Rightarrow) \quad I_5 &= \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3} + \frac{V_s}{R_4} \\
(\Leftarrow) \quad I_5 &= V_s \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right) = \frac{1}{R_{eq}}
\end{align*} \]

where \( R_1 + R_2 + R_3 + R_4 \).

Apply KCL at the top node gives

\[ +I_5 - I_1 - I_2 - I_3 - I_4 = 0 \]

\[ (\Rightarrow) \quad I_5 = I_1 + I_2 + I_3 + I_4. \]

\[ (\Leftarrow) \quad I_5 = \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3} + \frac{V_s}{R_4} \]

The equivalent resistance of \( N \) resistors connected in parallel is:

\[ \frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i} \]

Then the above circuit can be written as:

\[ \begin{align*}
I_5 \rightarrow \quad V_s \quad \Rightarrow \quad R_{eq}
\end{align*} \]
2-resistors in parallel:

Common prob:

\[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \]

\[ \Rightarrow \frac{R_1 R_2}{R_{eq}} = R_2 + R_1 \]

\[ \Rightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \]

Ex: Find \( R_{eq} \):

\( R_{eq} = R_{series} + R_{parallel} \)

\[ = 1\Omega + 3\Omega + \frac{2}{8} \]

\[ = 4 \Omega + \frac{2}{8} = \frac{8\Omega}{8} \]
Ex: Find $R_{eq}$.

\[
\begin{align*}
10\Omega & \parallel 10\Omega = \frac{10 \times 10}{10 + 10} = 5\Omega \\
20\Omega & \parallel 20\Omega = 10\Omega
\end{align*}
\]

\[R_{eq} = (5 + 10 + 15)\Omega = 30\Omega\]

**Voltage Division**

Voltage division often refers to a 2-resistor circuit, as shown here:

Say we want $V_1$ or $V_2$. Using $\text{KVL}$ and Ohm's law gives:

\[
V_S = V_1 - V_2
\]

\[
= V_1 + isR_2
\]

\[
= V_1 + \frac{V_1}{R_1}R_2
\]

\[
= V_1 \left( \frac{R_1 + R_2}{R_1} \right)
\]

\[
\Rightarrow V_1 = V_S \frac{R_1}{R_1 + R_2}
\]
Current Division

In a similar manner we can relate the branch currents in a 2-resistor network, as shown below,

\[ i = i_1 + i_2 \]

KCL & Ohm's Law gives:

\[ i = i_1 + \frac{V}{R_2} \]

\[ = i_1 + \frac{i_1 R_1}{R_2} \]

\[ = i_1 \left( \frac{R_2 + R_1}{R_2} \right) \]

\[ \Rightarrow i = i_1 \left( \frac{R_2}{R_1 + R_2} \right) \]

Power Computations

Known \( i + R \):

\[ V_U \]

\[ V_K \leq R \]

We know the power absorbed by \( R \) is \( P_A = V_R i \).

We also know \( V_K = i \times R \) (Ohm's Law), \( \Rightarrow P_A = i \times R \times i \)

\[ = i^2 R \]

Known \( V_0 + R \):

Again \( P = V_0 i \), we know \( i = \frac{V_R}{P} \), \( \Rightarrow P = V_R i \)

\[ = V_R \left( \frac{V_K}{R} \right) = \frac{V^2}{R} \]
Nodal Analysis

Works for all circuits, first illustrated with an example.

Consider:

\[ \begin{array}{c}
\text{Step 1: Define reference node. Typically here.} \\
\text{Step 2: Define nodes. Denoted as } N_1 + N_2 \text{ above.} \\
\text{Step 3: Write nodal voltages w.r.t. the ref. node. Denoted } V_1 + V_2 \text{ above.} \\
\text{Step 4: Denote currents in ekt.} \\
\text{Step 5: Write node voltage eqns. & solve:} \\
\end{array} \]

\[ + \left( \frac{V - V_1}{R_1} \right) - \frac{V_1}{R_2} - \frac{V_1 - V_2}{R_3} = 0 \]

and

\[ \left( \frac{V_1 - V_2}{R_3} \right) - \frac{V_2}{R_4} + 1 \_S = 0 \]

Now let \( V = 10 \), \( R_1 = 1 \), \( R_2 = 5 \), \( R_3 = 2 \), \( R_4 = 10 + i_s = 2A \).

For (1):
\[ \frac{10 - V_1}{1} - \frac{V_1}{5} - \frac{V_1 - V_2}{2} = 0 \]

For (2):
\[ \frac{V_1 - V_2}{2} + 2 = \frac{V_2}{10} \]
Solve for $V_1$ in 0:

$$10 - V_1 = \frac{V_1}{5} + \frac{V_1}{2} - \frac{V_2}{2}$$

$\Rightarrow 10 - V_1 - \frac{V_1}{5} - \frac{V_1}{2} = -\frac{V_2}{2}$

$\Rightarrow 10 - V_1(1 + \frac{1}{5} + \frac{1}{2}) = -\frac{V_2}{2}$

$\Rightarrow 10 - \frac{17}{10} V_1 = -\frac{V_2}{2}$

$\Rightarrow -\frac{17}{10} V_1 = -\frac{V_2}{2} - 10$

$\Rightarrow V_1 = \frac{10}{17} \left( \frac{V_2}{2} + 10 \right) \Rightarrow \text{sub into } 0$

$$\Rightarrow 10 \left( \frac{\frac{10}{17} \left( \frac{V_2}{2} + 10 \right)}{2} - \frac{V_2}{2} \right) = \left( \frac{V_2}{10} \right)^{10}$$

$\Rightarrow \frac{50}{17} \left( \frac{V_2}{2} + 10 \right) - 5V_2 + 20 = V_2$

$\Rightarrow \frac{25}{17} V_2 + 500 = 5V_2 + 20 = V_2$

$\Rightarrow V_2 \left( \frac{25}{17} - 5 \right) = -\frac{500}{17} - 20$

$\Rightarrow V_2 = 10.9 V$

$\Rightarrow V_1 = \frac{10}{17} \left( \frac{10.9}{2} + 10 \right)$

$\Rightarrow V_1 = 9.08 V$
Ex: Find \( V_1 + V_2 \)

\[ V_1 \quad 10k \quad V_2 \]

\[ 10k \quad y \]

\[ V_1 \quad 10k \quad V_2 \]

\[ 10k \quad y \]

\[ V_1 \quad 10k \quad V_2 \]

\[ 10k \quad y \]

\[ i_1 \quad i_2 \quad i_3 \]

\[ i_4 \quad i_5 \]

\[ N_i: \quad -i_1 + i_2 = i_3 = 0 \]

\[ \Rightarrow \quad i_3 = i_1 + i_2 \]

\[ N_o: \quad i_3 - i_4 - i_5 = 0 \]

\[ \Rightarrow \quad i_3 = i_4 + i_5 \]

\[ \Rightarrow \quad V_i/10k = \partial i/\partial i + \frac{U_3}{10k} \]

\[ y_{mA} = \frac{V_1}{10k} + \frac{V_i}{10k} \]

\[ \text{solve for } V_1 \]

\[ y_{mA} = V_1 \left( \frac{1}{10k} + \frac{1}{10k} \right) - V_2 \left( \frac{1}{10k} \right) \]

\[ \Rightarrow \quad V_1 = \frac{y_{mA} + V_2 \left( \frac{1}{10k} \right)}{2y_{10k}} \]

\[ = 20 + \frac{1}{2} V_2 \]

\[ \Rightarrow \quad 20 + \frac{1}{2} V_2 - V_2 = 40 + V_2 + V_2 \]

\[ \Rightarrow \quad 20 - V_2 = V_2 \left( 1 + \frac{1}{2} - \frac{5}{2} \right) \]

\[ \Rightarrow \quad -20 \cdot \frac{2}{5} = V_2 \]

\[ \Rightarrow \quad \boxed{V_2 = -8V} \]

\[ \Rightarrow \quad V_i = 20 + \frac{1}{2} \left( -8 \right) = 16V \]
\[ V_x = V_1 \]
\[ V_x = \frac{V_x}{6000} \]

\[ i_1 = i_2 + i_3 \]
\[ \Rightarrow 2mA = \frac{V_1}{3k} + \frac{V_x}{6000} \]
\[ V_1 = V_x \]
\[ \Rightarrow 2mA = \frac{V_x}{3k} + \frac{V_x}{6k} \]
\[ \Rightarrow V_x = 4 \]

\[ \frac{V_x}{6000} = \frac{V_3}{12k} + \frac{V_2}{12k} \]
\[ \Rightarrow V_3 = V_0 \]
\[ \Rightarrow \frac{V_3}{60k} = V_0 \left( \frac{1}{12k} + \frac{1}{12k} \right) \]
\[ \Rightarrow V_0 = \frac{2 \cdot 4}{2} \]
\[ = [4V] \]

**Supernodes**

*If a voltage exists between two nodes that are neither the reference node, we can define a supernode & sum eqns.*
Consider:

Find \( I_o \):

We know \( V_i = 6V \) and \( V_y = -4V \). Treat supernode as one node and sum currents:

\[
-i_1 - i_2 - i_3 - i_4 = 0
\]

\[
i_1 = \frac{V_3 - V_i}{2k} \quad i_2 = \frac{V_2}{1k} \quad i_3 = \frac{V_3}{2k} \quad i_4 = \frac{V_3 - V_y}{2k}
\]

\[
\left( \frac{V_3 - V_i}{2k} \right) - \frac{V_2}{1k} - \frac{V_3}{2k} - \frac{V_3 - V_y}{2k} = 0
\]

\[V_i = 6V, \ V_y = -4V\]

\[
(\frac{V_3 - 6}{2k}) - \frac{V_2}{1k} - \frac{V_3}{2k} - \frac{V_3 + 4V}{2k} = 0
\]

Also \( V_3 - V_o = 4V \)

\[
\Rightarrow V_3 = 12V + V_2
\]

\[
\Rightarrow -\left( \frac{V_3 - 6}{2k} \right) - \frac{V_2}{1k} - \frac{12 + V_3}{2k} - \frac{12 + V_3 + 4}{2k}
\]
Solve for $V_3$:

\[-\frac{V_3}{2k} + \frac{6}{2k} - \frac{V_2}{1k} - \frac{12}{2k} = \frac{12}{2k} + \frac{V_5}{2k} + \frac{4}{2k}\]

\[\Rightarrow V_3 \left(\frac{-1}{2k} - \frac{1}{1k} = \frac{1}{2k} - \frac{1}{2k}\right) = \frac{-6}{2k} + \frac{12}{2k} + \frac{10}{2k} + \frac{4}{2k}\]

\[\Rightarrow V_3 = \frac{0.011}{-2.5 \times 10^{-3}}\]

\[= -4.4V\]

\[\Rightarrow V_3 = 12 - 4.4V = 7.6V\]

\[\Rightarrow I_0 = \frac{V_3}{2k} = \boxed{3.8mA}\]

**Mesh Analysis**

Mesh analysis is another general technique (similar to nodal) based on KVL. Note that nodal is based on KCL.

Let's introduce the idea with an example.

Consider:

[Diagram of the circuit with nodes and voltages labeled]
**Step 1:** Label all voltages

**Step 2:** Define loops $L_1, L_2, \ldots, L_n$

**Step 3:** Define loop currents $I_1, I_2, \ldots, I_n$

**Step 4:** Write KVL eqns around each loop.

**Step 5:** Write voltages in terms of loop currents.

\[ L_1: \quad V_1 + V_3 + V_2 - V_a = 0 \quad V_5 + V_4 + V_5 - V_3 = 0 \]

From Ohm's law:

\[ V_1 = i_1 R_1, \quad V_3 = i_1 R_3, \quad V_5 = (i_1 - i_2) R_3, \quad V_4 = i_2 R_4 + V_5 = l_2 R_5 \]

Sub into $L_1 + L_2$ eqns.

\[ \Rightarrow i_1 R_1 + (i_1 - i_2) R_3 + i_1 R_2 = V_a \quad \& \quad V_5 + l_2 R_4 + l_2 R_5 = V_3 \]

**Ex:**

\[ L_1: 12V = (6k) i_1 + 6k(l_1 - i_2) \]  
\[ L_2: -3V = -6k(i_1 - i_2) + 3i_2 \]

Solve for $i_1$:

\[ 12 = i_1 (6k + 6k) - 6ki_2 \]

\[ \Rightarrow \frac{12 + 6ki_2}{12k} = i_1 \]
1 \times 10^{-3} + 0.5 \dot{i}_{3} = \dot{i}_{1} \quad \text{sub \ 1 \text{mA}}

\Rightarrow -3V = -6k \left( 1 \times 10^{-3} + 0.5 \dot{i}_{3} - \dot{i}_{2} \right) + \dot{i}_{2} 3k

\Rightarrow -3V = -6 - 3\dot{i}_{0} + 6k\dot{i}_{2} + \dot{i}_{2} 3k

\Rightarrow 3V = 6k\dot{i}_{2} \Rightarrow \dot{i}_{2} = 0.5mA.

\Rightarrow \dot{i}_{1} = 1mA + 0.5 \times 0.5mA = 1.25mA.

**Thévenin & Norton Equivalents**

*Thévenin equivalent ckt*: A series connected independent voltage source ($V_{TH}$) + resistance ($R_{TH}$) used to model a ckt connected between two terminals, say a + b. or

\[ V_{TH} \]

\[ R_{TH} \]

\[ \text{a} \]

\[ \text{b} \]

**Intro through an example:**

Consider:

\[ 3V \]

\[ 2k \]

\[ 1k \]

\[ 6k \]

\[ V_{0} \]
For circuits w/ only independent sources:

1) Remove the resistor w/ \( V_0 \) & compute \( V_{oc} \).

\[
\begin{align*}
V_{oc} &= 2\text{mA} \cdot 3k + 3V = 9V
\end{align*}
\]

2) Zero-out the sources by short circuiting the voltages & open circuiting the current sources:

\[
R_{T_{in}} = 2k + 1k = 3k
\]

3) Redraw circuit attached to resistor of interest:

\[
V_{oc} = 9V \cdot \frac{6k}{9k} = 6V
\]
Norton equivalent circuit: Similar to Thevenin, except $R_{th}$ is connected in parallel w/ $i_{th}$, or a current source.

For circuits w/ only independent sources:

1) Remove the resistor w/ $V_o$ & short the terminals to find $i_{sc}$, or

\[ i_i = \frac{3V}{1k + 2k} = 1mA \quad \Rightarrow \quad i_{sc} = 3mA \]

2) Same as Thevenin circuit process to find $R_{th}$.

\[ R_{th} = 3k \]

3) Draw Norton equivalent:

\[ 3mA \quad 3k \quad 6k \quad V_o \]
D4.5 Find $V_o$ in the following network using Thévenin's theorem.

**ANSWER:** $12V$

Let us use Thévenin's theorem to find $V_o$ in the network in Fig. 4.8a, which is redrawn in Fig. 4.12a.

**SOLUTION** If we break the network to the left of the current source, the open-circuit voltage $V_{oc}$ is as shown in Fig. 4.12b. Since there is no current in the 2-kΩ resistor and therefore no voltage across it, $V_{oc}$ is equal to the voltage across the 6-kΩ resistor, which can be determined by voltage division as
\[ V_{oc} = 12 \left( \frac{6k}{6k + 3k} \right) = 8 \text{ V} \]

The Thévenin equivalent resistance, \( R_{Th1} \), is found from Fig. 4.12c as

\[ R_{Th1} = 2k + \frac{(3k)(6k)}{3k + 6k} = 4 \text{ k}\Omega \]

Connecting this Thévenin equivalent back to the original network produces the circuit shown in Fig. 4.12d. We can now apply Thévenin’s theorem again, and this time we break the network to the right of the current source as shown in Fig. 4.12e. In this case \( V_{oc} \) is

\[ V_{oc} = (2 \times 10^{-3})(4k) + 8 = 16 \text{ V} \]

and \( R_{Th2} \) obtained from Fig. 4.12f is 4 k\( \Omega \). Connecting this Thévenin equivalent to the remainder of the network produces the circuit shown in Fig. 4.12g. Simple voltage division applied to this final network yields \( V_o = 8 \text{ V} \). Norton’s theorem can be applied in a similar manner to solve this network; however, we save that solution as an exercise for the reader.

---

**Circuits with both Dependent & Independent Sources:**

For these cases, we need to compute \( V_{oc} + I_{sc} \).

Then \( R_{Th} = \frac{V_{oc}}{I_{sc}} \).

Note that we need to choose a point to remove the resistor of interest such that any dependent voltage or currents remain in the circuit.

Let’s look at an example:
**EXAMPLE 4.12**

Let us use Thévenin’s theorem to find $V_o$ in the network in Fig. 4.16a.

![Network Diagram](image)

**Figure 4.16** Circuits used in Example 4.12.

**SOLUTION** To begin, we break the network at points A-B. Could we break it just to the right of the 12-V source? No! Why? The open-circuit voltage is calculated from the network in Fig. 4.16b. Note that we now use the source $2000I_x$ because...
this circuit is different from that in Fig. 4.16a. KCL for the supernode around the 12-V source is

\[
\frac{(V_{oc} + 12) - (-2000I_x')}{1k} + \frac{V_{oc} + 12}{2k} + \frac{V_{oc}}{2k} = 0
\]

where

\[I_x' = \frac{V_{oc}}{2k}\]

yielding \(V_{oc} = -6\) V.

\(I_x\) can be calculated from the circuit in Fig. 4.16c. Note that the presence of the short circuit forces \(I_x'\) to zero and, therefore, the network is reduced to that shown in Fig. 4.16d.

Therefore,

\[I_x = \frac{12}{2k} = -18\text{ mA}\]

Then

\[R_{th} = \frac{V_{oc}}{I_x} = \frac{1}{3}\text{ k}\Omega\]

Connecting the Thévenin equivalent circuit to the remainder of the network at terminals A-B produces the circuit in Fig. 4.16e. At this point, simple voltage division yields

\[V_o = (-6) \left(\frac{1k}{1k + 1k + \frac{1}{3}k}\right) = \frac{-18}{7}\text{ V}\]

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**Super Position**

For a given ckt. with only independent sources, we can apply each src individually & add the results of each of the sources for the total response.

Consider:

![Circuit Diagram](image-url)
Step 1: Turn off all sources except the 1st one.


It can be shown $i_1 = 15A$, $i_2 = 10A + i_3 = 5A = i_4$

Step 3: Repeat steps 1 & 2 for sources

$\Rightarrow i_1'' = 2A$, $i_2'' = -9A$, $i_3'' = 6A + i_4'' = -6A$

$\Rightarrow i_1 = i_1'' + 1'' = 17A$, $i_2 = i_2'' + i_3'' = 6A$

$i_3 = i_3'' + i_3 = 11A$, $i_4 = i_4'' + i_4 = -1A$