Analytic Expressions for Small Loop Antennas-With Application to EMC and RFID Systems

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Topics

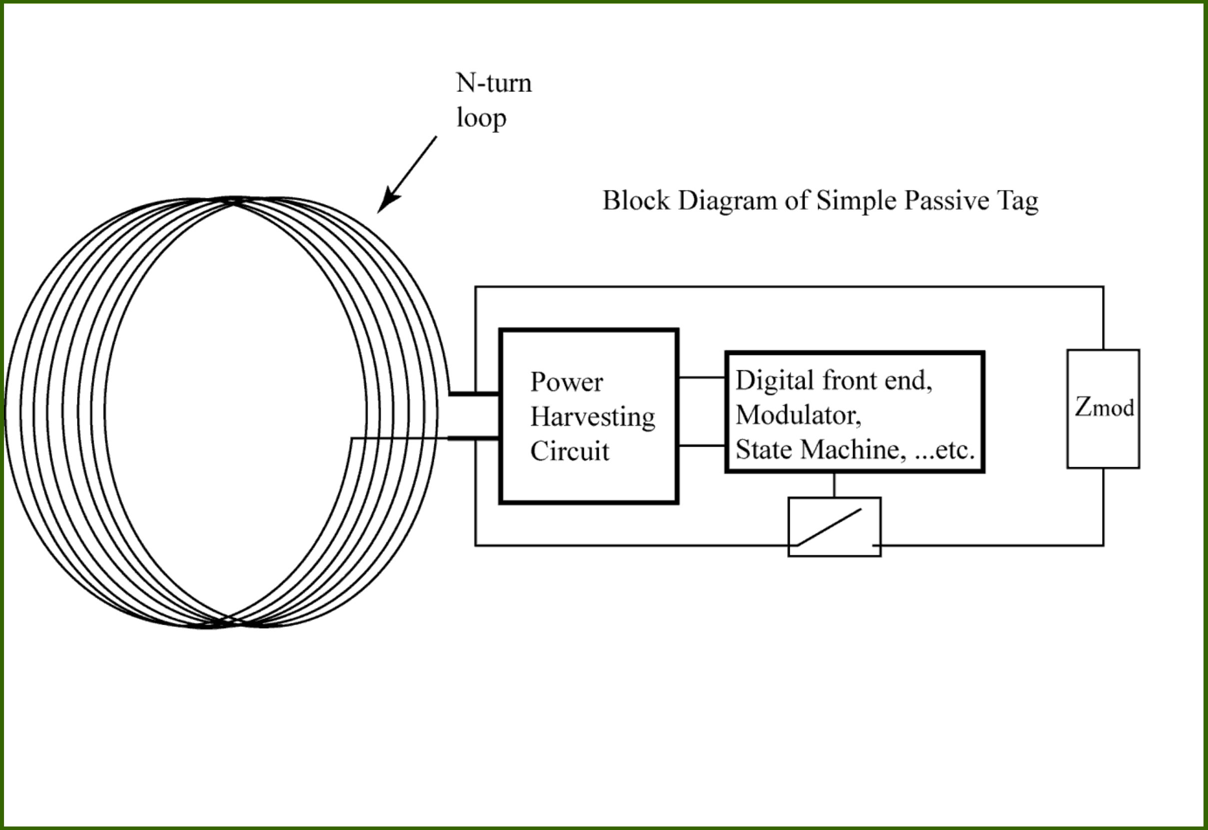
- Introduction.
- Derivation of analytical expressions.
- Verification of analytical expressions.
- Results
  - Induced currents on a loop from incident field
  - Induced currents on a loop from another loop.
- Conclusion.

Introduction

- A new way of thinking about the operation of inductively coupled RFID systems
- Loops are used in may applications
  - “sniffer” probes
  - Inductively couple energy to implantable devices [1]-[5]
  - Wireless communication
  - RFID [6]
- Reader→transponder(tag) via EM waves.
- Harvests energy and communicates.

Introduction

Low frequency RFID Tag

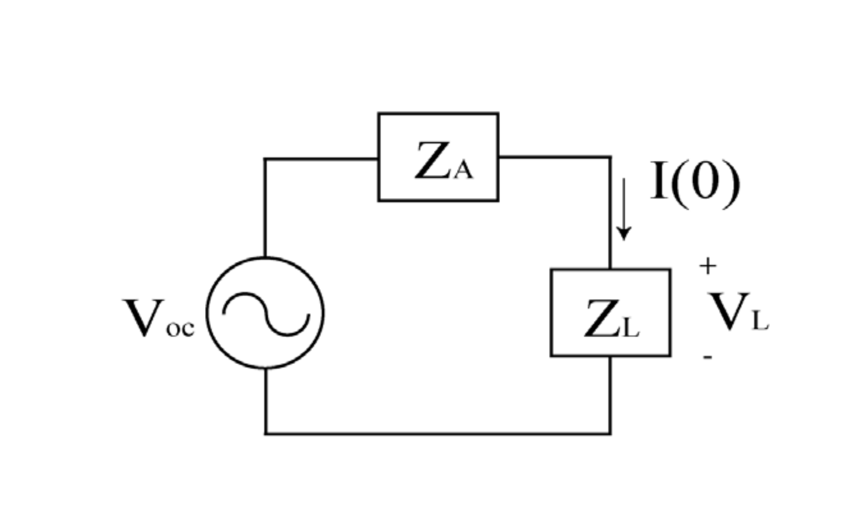


Introduction

- We investigate the induced current in a small loop as well as the magnetic field created by that induced current as well as the load effect.
- Two sources
  - Incident field
  - Other small loop
- Verified with MININEC [7]

Derivation of analytical expressions

Consider the following receiving antenna equivalent circuit [8]-[9]:



When the radius of the loop is less than .03λ [9] we can assume the current is constant in the loop. This then gives:

$$I(0) = \frac{V_{oc}}{Z_A + Z_L} \qquad (1)$$

Derivation of analytical expressions

The voltage developed across the open circuit terminals of a loop antenna is related via Faraday's law to a time-varying magnetic flux cutting through the loop [8]-[10].

For an N-turn loop Faraday's law gives the phasor-domain voltage:

$$V_{oc} = -j\omega N \iint \overline{B} \bullet d\overline{s} \qquad (2)$$

where ω is the radian frequency, $\overline{B}$ is the magnetic flux density, and $d\overline{s}$ is the incremental surface area of the loop.

Derivation of analytical expressions

Since we assumed a small loop size we can approximate the magnetic flux crossing the loop as:

$$\iint \overline{B} \bullet d\overline{s} \approx B\pi b^2 \tag{3}$$

where $B = |\overline{B}|$ and b is the radius of the loop. This then gives:

$$V_{oc} = -j\omega N\pi b^2 \mu_0 H \tag{4}$$

where $B = \mu_0 H$.

If desired, the voltage across the load Z_L can be obtained from V_{oc} as

$$V_L = V_{oc} \frac{Z_L}{Z_L + Z_a} \tag{5}$$

Derivation of analytical expressions

To determine the current induced in the loop we need to know the input impedance of the loop antenna. In general this impedance is expressed as:

$$Z_a = R_a + jX_a \qquad (6)$$

where R_a and X_a are the input resistance and reactance of the antenna, respectively.

The input resistance is typically modeled as having two components,

$$R_a = R_r + R_\Omega \qquad (7)$$

where R_r is the radiation resistance of the antenna and R_Ω is the ohmic (or loss) resistance of the antenna.

Derivation of analytical expressions

The radiation resistance of a small, N-turn loop is approximately [8]:

$$R_r = 31{,}200\left(\frac{N\pi b^2}{\lambda^2}\right)^2 \qquad (8)$$

and the ohmic loss for each turn can be accounted for by

$$R_\Omega = \left(\frac{b}{a}\right)\sqrt{\frac{\omega\mu_0}{\sigma}} \qquad (9)$$

for a wire with conductivity σ, loop radius b, and wire radius a.

Derivation of analytical expressions

Small loops tend to be very inductive, with inductance of an N-turn loop typically approximated as [12]

$$L = N^2 \mu_0 b \left(\ln\left(\frac{8b}{a}\right) - 2 \right) \qquad (10)$$

The reactance of a small loop tends to be larger [13]. This assumption is made in this work.

Derivation of analytical expressions

This then gives the input impedance of a N-turn small loop as approximately:

$$Z_a = j\omega\mu_0 bN^2\left(\ln\left(\frac{8b}{a}\right)-2\right) \qquad (11)$$

Using (11) and the expression for the open circuit voltage in (4) we can express the induced current in a small N-turn loop as:

$$I = \frac{-j\omega N\pi b^2\mu_0 H}{j\omega\mu_0 bN^2\left(\ln\left(\frac{8b}{a}\right)-2\right)+Z_L} \qquad (12)$$

← Notice

Derivation of analytical expressions

If the current I flowing in a small loop of radius b centered at the origin in the x-y plane is known the electric and magnetic fields created by the current can be determined by the following expressions [8]-[11]:

$$H_r = \frac{j\beta b^2 I \cos\theta}{2r^2} e^{-j\beta r} \left\{ 1 + \frac{1}{j\beta r} \right\} \quad (13)$$

$$H_\theta = \frac{-(\beta b)^2 I \sin\theta}{4r} e^{-j\beta r} \left\{ 1 + \frac{1}{j\beta r} - \left(\frac{1}{\beta r} \right)^2 \right\} \quad (14)$$

$$E_\phi = \frac{\eta (\beta b)^2 I \sin\theta}{4r} e^{-j\beta r} \left\{ 1 + \frac{1}{j\beta r} \right\} \quad (15)$$

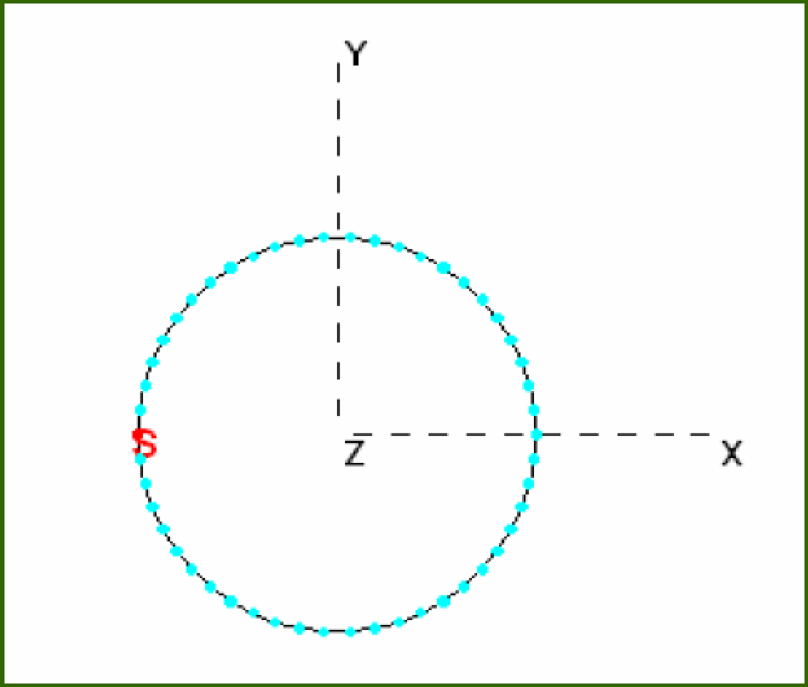
where η is the intrinsic impedance of the medium through which the wave is propagating.

Derivation of analytical expressions

- First we will consider the situation when the loop antenna is far from the source then we will consider the situation when the incident magnetic field is another loop lying in close proximity of the receiving loop.
- In an effort to think of RFID systems from an induced-current/scattered-field perspective we determine the induced current and scattered field for two different load impedances.
- It will be clearly shown that the “reader” can detect what happens at the “tag” when the load of the tag changes.

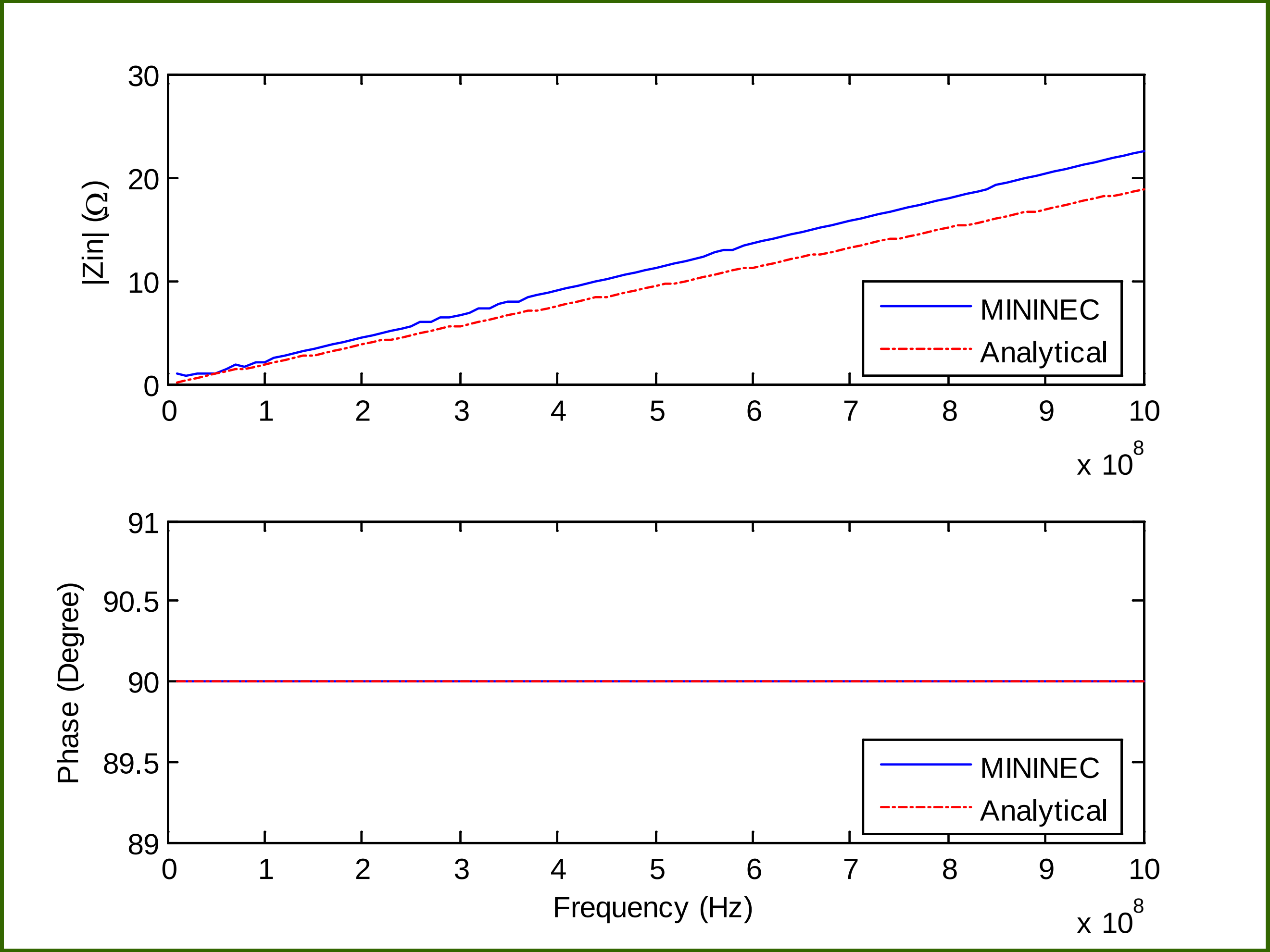
Verification of analytical expressions

Consider the small single-turn loop with a loop radius of 1mm. This loop was modeled in MININEC with 50 segments. For each segment the wire radius was .1mm. A 1A current source was used to excite the problem.



The following is the magnitude and angle of the input impedance.

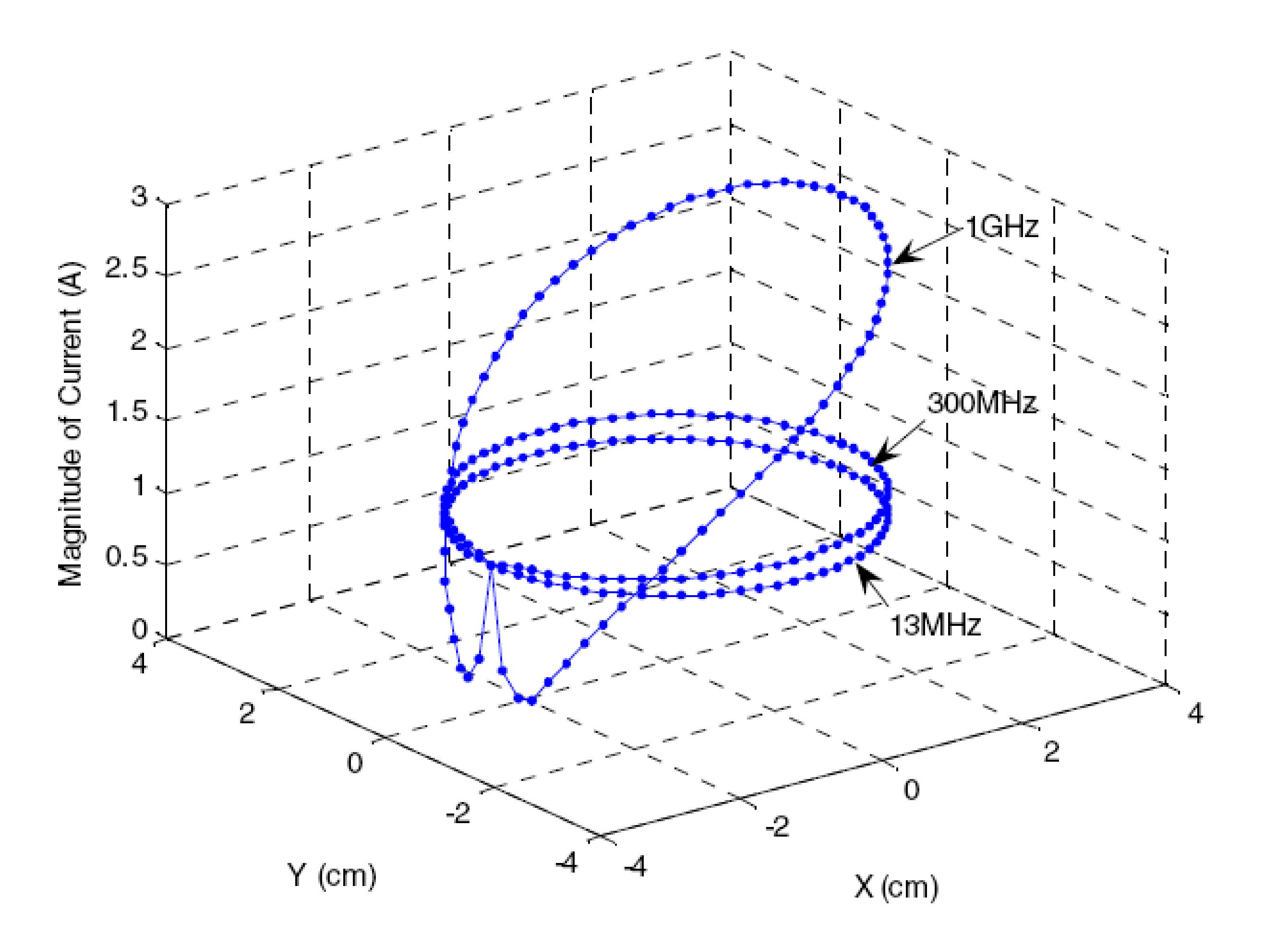
Verification of analytical expressions



Verification of analytical expressions

- Calculation of the input impedance with MININEC verified that the assumption of constant current distribution used in deriving the expression for the input impedance of a loop is fairly reasonable for the given simulation frequency range (10 MHz to 1 GHz).
- To determine the effect the loop size has on the current, a loop with a loop radius of 2.5cm was evaluated in MININEC
- The wire radius was 0.1 mm and the loop was driven with 1 A current source.
- The simulation was carried out for three frequencies (13 MHz, 300 MHz and 1 GHz).

Verification of analytical expressions



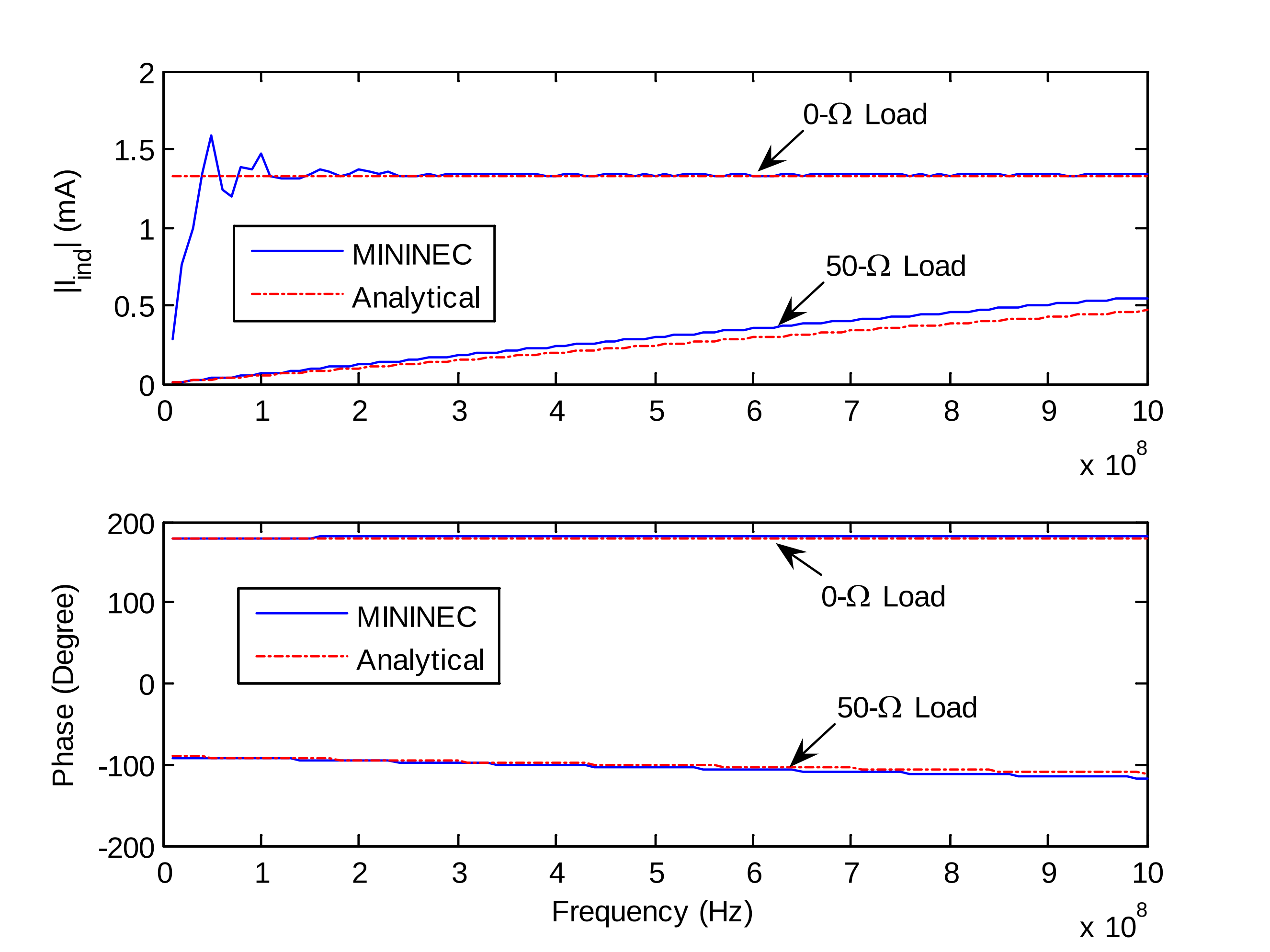
Verification of analytical expressions

- At 13 MHz $b = 0.001\lambda$ the current distribution is essentially constant around the loop.
- At 300 MHz $b = 0.025\lambda$ and the current varies slightly.
- At 1 GHz $b = 0.083\lambda$ and significant variation is noted.

Results

- Consider the same loop described above – i.e., a loop of 1 mm radius made from 1 turn of 0.1 mm wire – to investigate the induced current on a small loop as a result of an incident magnetic field.
- The intent of this work is to provide approximate closed-form expressions that can be used to describe the operation of inductive RFID systems from an induced-current/scatted-field perspective.
- As a check on accuracy, the results of our analytical expression for current is compared with those determined from MININEC when the incident magnetic field is 1 A/m.

Results



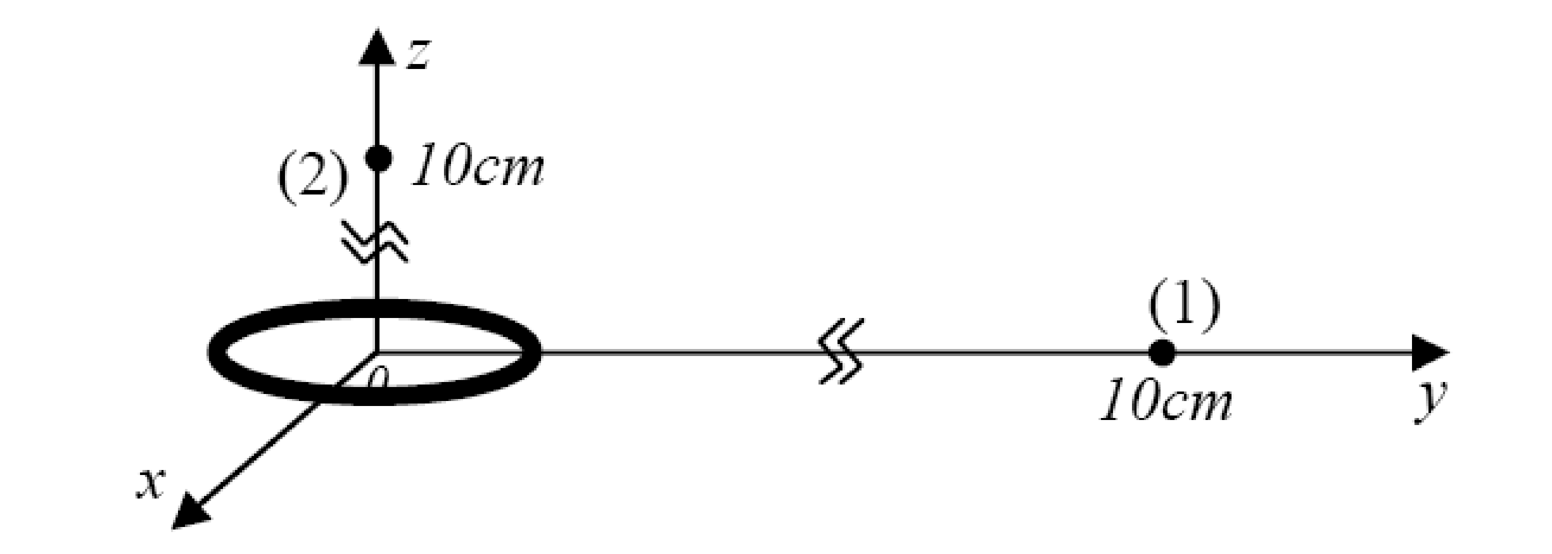
2
1.5
1
0.5
0
$|I_{ind}|$ (mA)
0-Ω Load
50-Ω Load
MININEC
Analytical
0
1
2
3
4
5
6
7
8
9
10
x 10^8
200
100
0
-100
-200
Phase (Degree)
0-Ω Load
50-Ω Load
MININEC
Analytical
Frequency (Hz)
x 10^8

Results

- Next we investigate the effect the previous change in current has on the scattered field.
- This is done by determining the scattered field for two locations, as shown in the following figure.

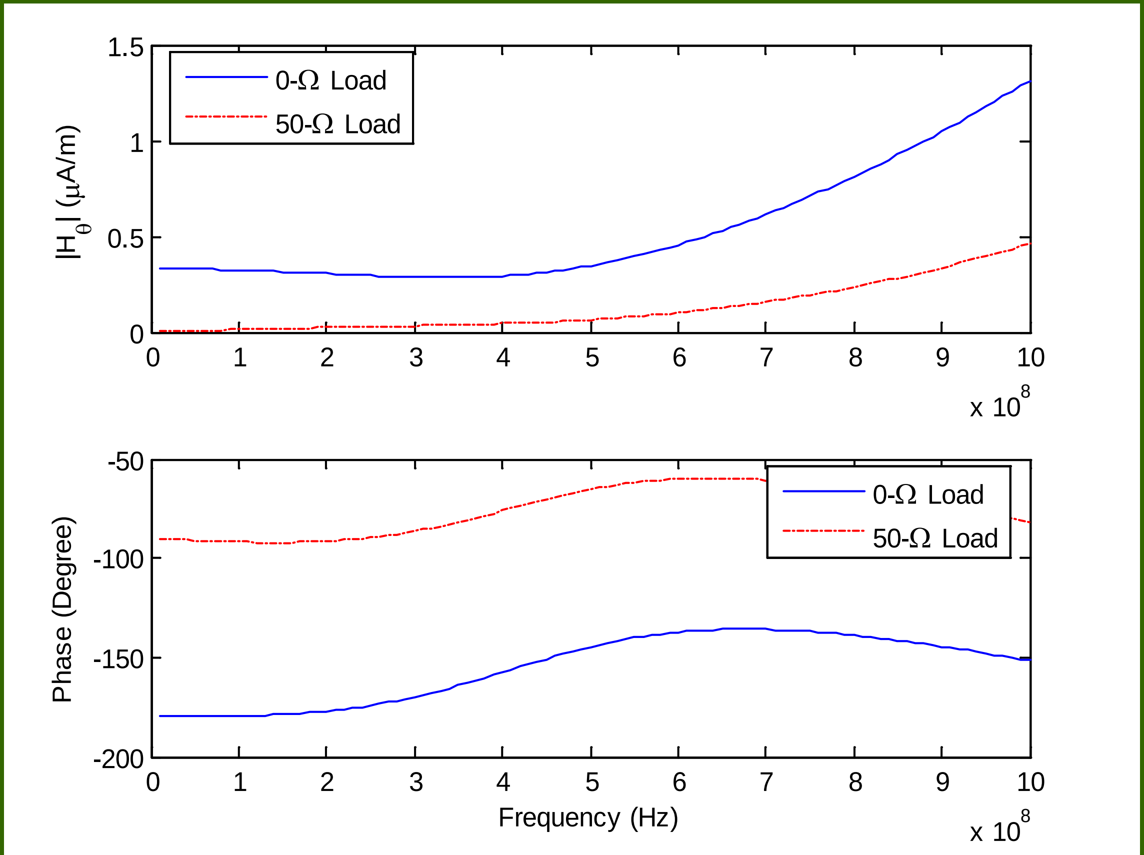
Results

- Position (1) models what scattered field might be expected in a "reader loop" located in the same plane as the "tag loop".
- Position (2) models what might be detected in a "reader loop" placed directly above or below the "tag loop".



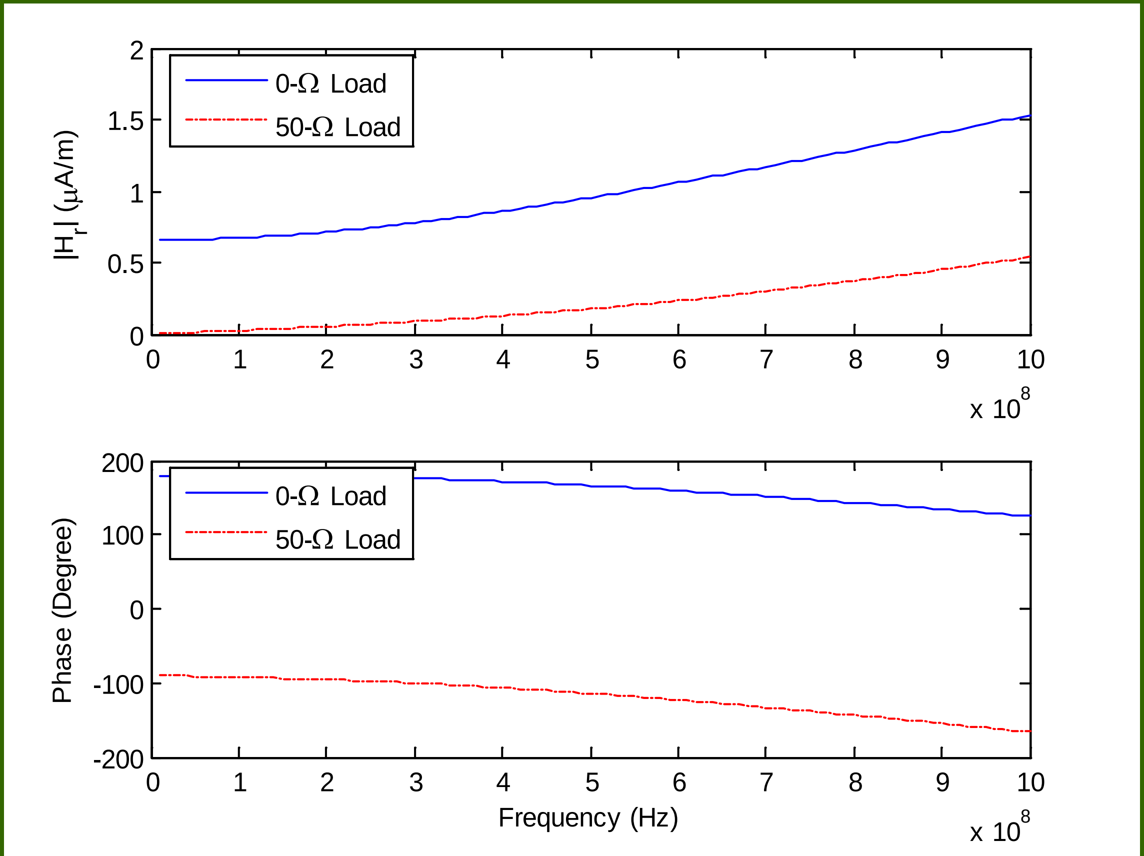
Results

Point (1) (same plane as loop)



Results

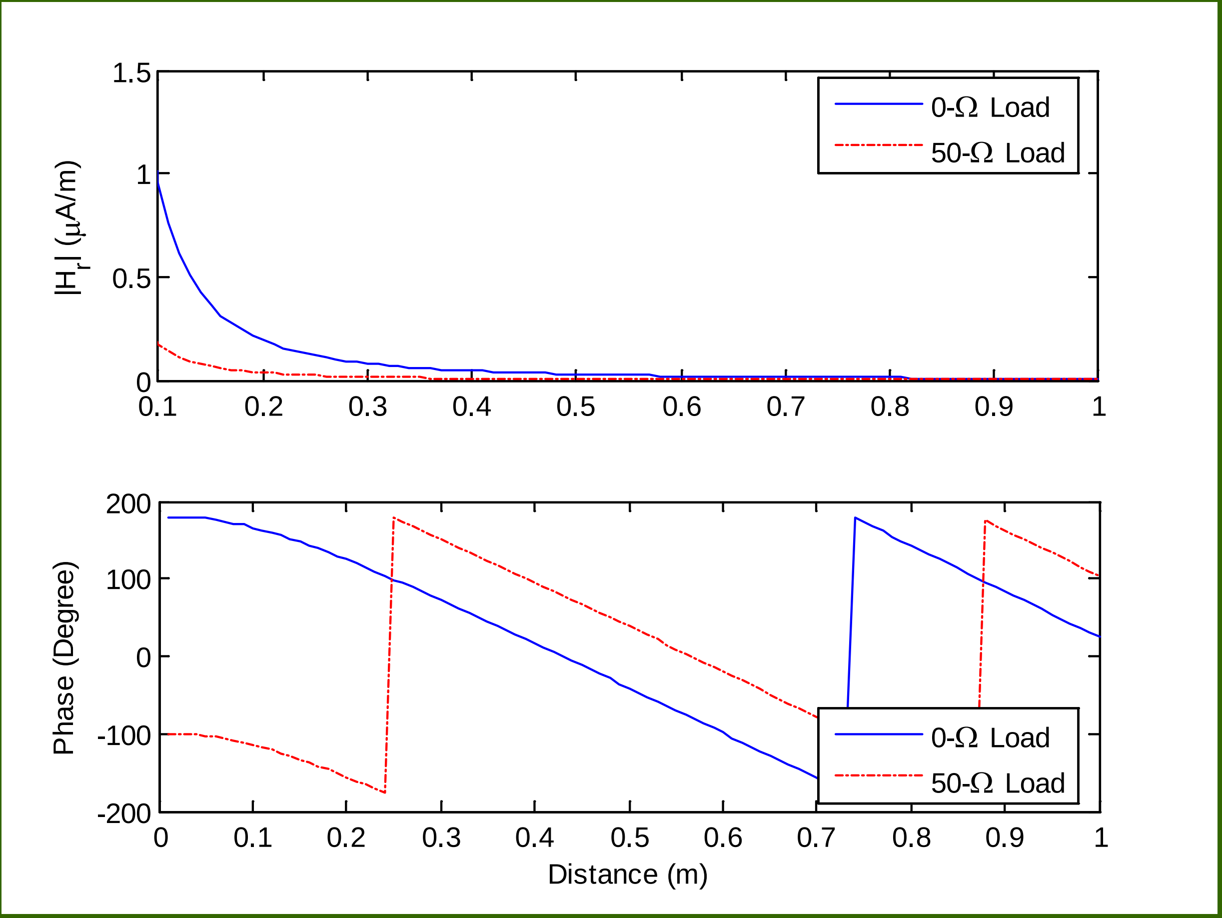
Point (2) (directly above loop)



Results

- We are also interested in observing how the scattered field varies with distance from the loop. This is illustrated in the following figure, where the scattered field above the “tag loop” is determined when the operating frequency is fixed at 500 MHz.
- This provides the important result that the ability to remotely distinguish what happens at a “tag” diminishes as one moves away from the tag.

Results



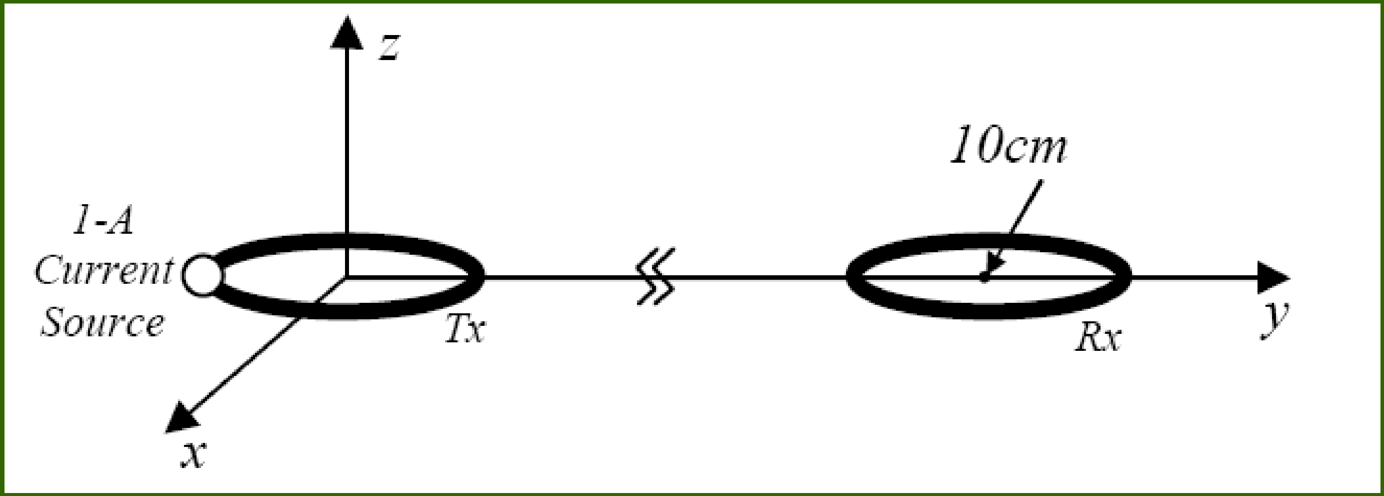
1.5
1
0.5
0
$|H_r|$ (μA/m)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0-Ω Load
50-Ω Load
200
100
0
-100
-200
Phase (Degree)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Distance (m)
0-Ω Load
50-Ω Load

Results

- Now suppose one has a reader that is inductively coupled with the tag.
- Again assume both loops are small enough such that a constant current exists.
- This then allows us to determine the induced current on the tag antenna as well as the induced current on the reader antenna as a result of the scattered field from the tag.

Results

- This method can be used for any loop position.
- In this (initial) work we provide the results for one particular case.
- The scenario under investigation is shown below



Results

Assuming that the original current in the reader is I_0 and the loops have center-to-center spacing distance *d* then the scattered field back at the transmitter (i.e., reader) can be written as:

$$H_{\theta s} = \frac{-j\omega\pi\beta^4 b^6 \mu_0 I_0 \sin^2\theta}{16d^2\left[j\omega\mu_0 b\left(\ln\frac{8b}{a}-2\right)+Z_L\right]} e^{-2j\beta d}\left\{1+\frac{1}{j\beta d}-\left(\frac{1}{\beta d}\right)^2\right\}^2 \quad (16)$$

Results

This then results in an induced current at the reader of:

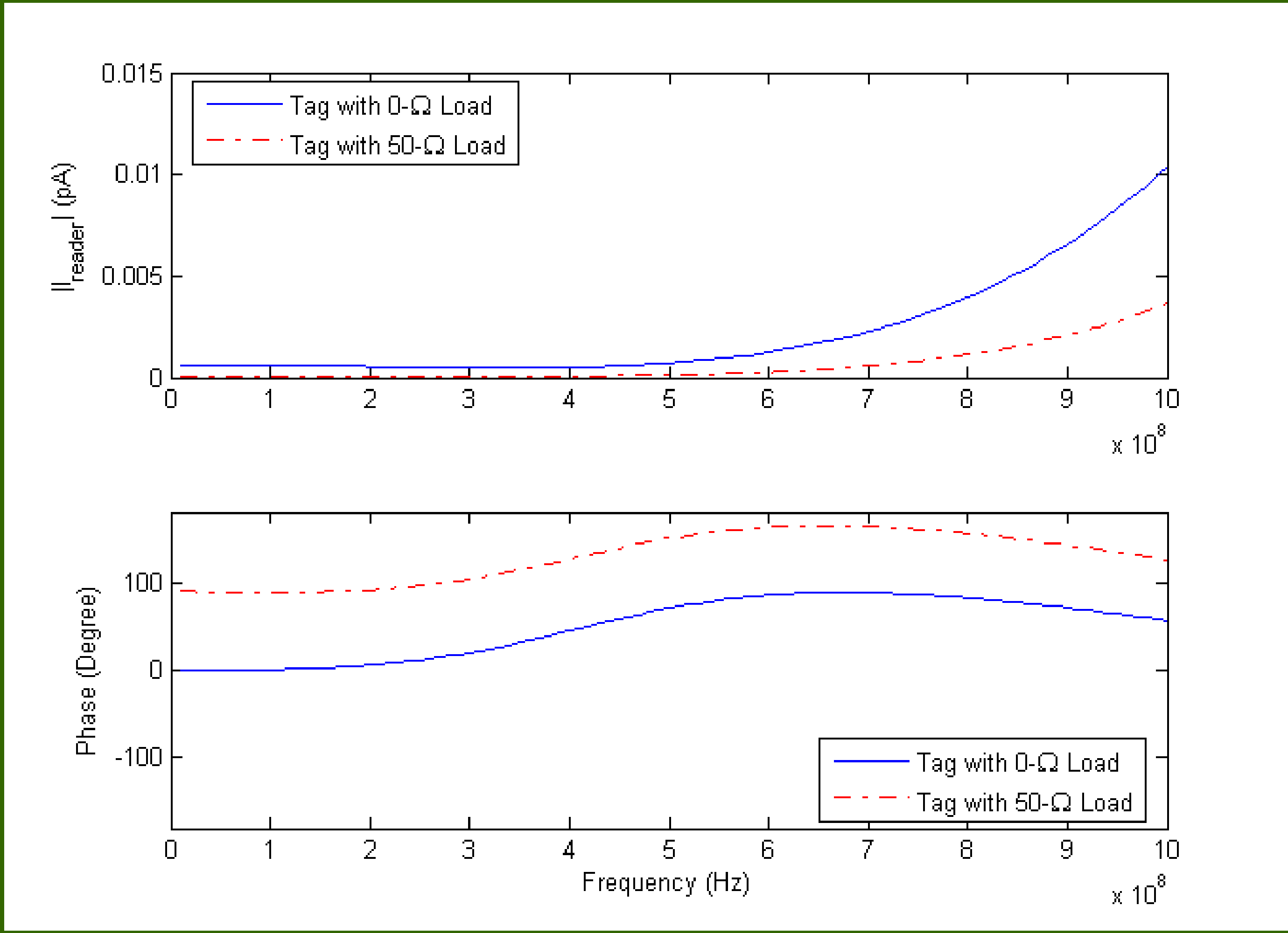
$$I_{reader} = \frac{-\omega^2 \pi^2 \beta^4 b^8 \mu_0{}^2 I_0 \sin^2 \theta}{32 d^2 \left[j\omega\mu_0 b \left(\ln \frac{8b}{a} - 2 \right) + Z_L \right] Z_{reader}} e^{-2j\beta d} \left\{ 1 + \frac{1}{j\beta d} - \left(\frac{1}{\beta d} \right)^2 \right\}^2$$

(17)

Results

- As a final numerical result, assume that the input impedance of the reader is 50Ω, I_0=1A, d=10 mm, and the two loops are in the same plane.
- This then gives the following induced current at the reader as a results of the scattered fields from the tag.

Results



0.015
0.01
0.005
0
$|I_{reader}|$ (pA)
Tag with 0-Ω Load
Tag with 50-Ω Load
0 1 2 3 4 5 6 7 8 9 10
x 10^8
100
0
-100
Phase (Degree)
Tag with 0-Ω Load
Tag with 50-Ω Load
0 1 2 3 4 5 6 7 8 9 10
Frequency (Hz)
x 10^8

Conclusion

- We investigated the current induced in a small loop as a results of
  - An incident magnetic field
  - Another small loop in close proximity
- We illustrated how the load on the tag antenna can effect the scattered field and hence the induced current on the reader
- Alternate way of thinking about RFID systems
- Useful for EMC engineers for estimating currents induced on small loops in RFID systems

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