Lecture 3

Fundamentals of Physics

Phys 120, Fall 2014

How things move

A. J. Wagner
North Dakota State University, Fargo, ND 58102

Fargo, September 1, 2015

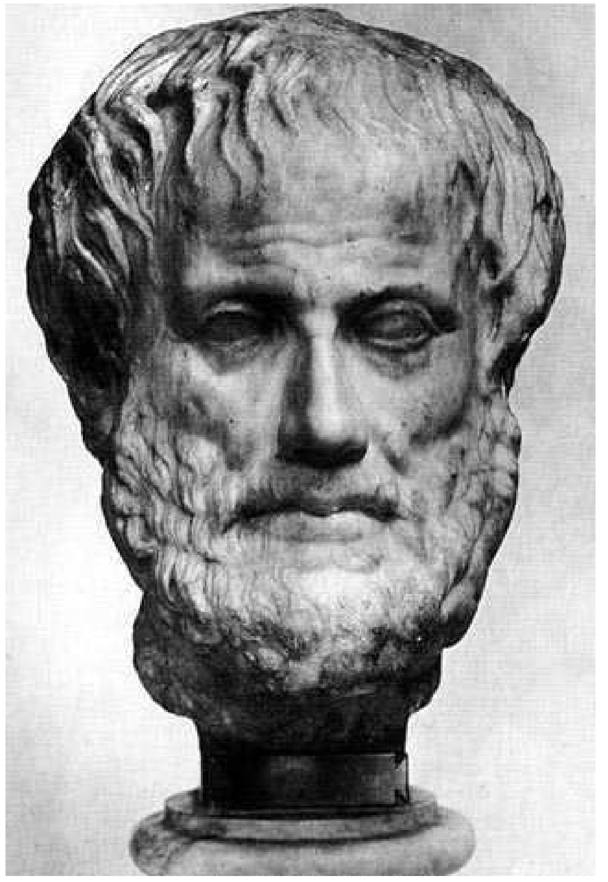
Overview

- Aristotle
- Galileo
- Law of inertia: basic of Newtonian Physics
- Describing motion: position, speed and velocity
- Acceleration (falling)
- Review

How do things change?

Aristotle (384-322 BC) developed a very general theory of change, which included a change from sickness to health, of change in color, and of change in position all on a very abstract basis.

Many of these are still poorly understood (e.g. moving from thickness to health), but the problem *change of position*, which is the basic of Physics, has evolved a lot. Aristotle identified three kinds of motion:



Natural motion: the motion of a falling "earthlike" object like a stone, or the rising of a "firelike" object like flames. Every element wants to move to its natural position.[a]

Violent motion: motion can be induced by an actor, like dragging a log sideways. Such motion will cease when no force is applied. (More difficulties arise in the description of the flight of an arrow. . .)

Celestial motion: In the heavens motion is perfect and perpetual and, once brought into motion by the "prime mover", will continue for ever.

[a]There are supposed to be four elements: earth, water, air, and fire in order of heavyness.

Observations leading to Aristotelian Physics

One of the first experiences that we have as babies is that we are able to manipulate objects and that we can make them move to a new position.

Then we discover that we are able to move ourselves.

Eventually we discover the exciting law of falling objects. If you let an object go, it will fall to the ground **every time**! This is so exciting that small children will continue to drop objects from their high-chair over and over and over again. They have discovered a law, an experiment with repeatable results. Once we have done this enough times, the amazing discovery that objects fall seems natural to us.

Much later we notice the amazing regularity of night and day, of the moon phases, of the seasons. After experiencing this enough times, we get used to this regularity as well.

The Aristotelian ideas agree well with these discoveries.

Difficulties with Aristotelian Physics

However, if you look at the flight of an arrow, the situation becomes murky. Its natural motion should be just falling to the ground. Once the arrow has left the bow, why does it keep going and does not immediately drop?

If you have a heavy object and a light one, the heavy object is attracted much more strongly to the ground, so should it not fall faster to the ground?

There were several attempts to overcome these difficulties*, but in the end a completely different description of motion had to be developed.

*like the theory of impetus by John Philoponus in the 6th century

From antiquity to the Renaissance

We discussed the progress of mapping the motion of the planets along their path. There was little progress in quantitatively increasing our understanding of motion on earth.

Quantitative understanding of motion developed only slowly.

Aristotle already described motion as an object changing position as function of time. He argued that a quicker object will move the same distance in less time, or it will move more distance in the same time. Here distance and time are the critical measures of motion, whereas "quickness" or speed never achieves that same status.

Gerard of Brussels, a mathematician who my have taught at the University of Paris in the first half of the thirteenth century wrote a brief *Book on Motion. A group of mathematicians and logicians affiliated with Merton College, Oxford developed these ideas of describing motion further.*

Merton school of motion (about 1330 CE)

They noted that motion can be described from two points of view: cause (dynamics) and effect (kinematics). They worked on the latter.
To describe motion they introduced the concepts of average velocity and instantaneous velocity.
Using these concepts they were able to describe mathematically uniform and uniformly accelerated motion (as we will see in a few moments).

Speed and velocity

If an object is moving at an unchanging speed, its position is changing at a constant rate. If you move with a speed of 1 m/s, then every second you will have moved another meter.

Average speed: If you drive 32 km in half an hour, your average speed is

$$\text{Average Speed} = \frac{\text{distance traveled}}{\text{time taken}} = \frac{32km}{0.5h} = 64\frac{km}{h}$$

Instantaneous speed: the same as average speed but examined for a very small time interval.

As a shorthand we can write

$$\bar{s} = \frac{d}{t}$$

Directions matter: speed with a direction is called **velocity**.

Acceleration

If there are outside forces acting on an object, it will change its velocity. We define this change as the **acceleration**.

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time to make the change}}$$

Important: there is not only acceleration when the speed of an object changes, but also when the direction changes!

Concept check

1. A speeding bullet moving through air.

2. A race car just as it begins to “dig out” from rest.

3. A fast train as it moves around a long and gentle curve.

4. A fast car as it hits a brick wall.

5. A golf ball at the instant it is stuck by a fast-moving golf club.

Which have high velocity and low acceleration?
Which have low velocity and high acceleration?

Results of the Merton School

As we saw uniform motion is motion where at equal time intervals equal amounts of space are covered, or more simply motion where the instantaneous velocity is constant. Here the distance traveled increases linearly with time.

The next simple motion is uniformly accelerated motion, or motion where the instantaneous velocity increases linearly in time. For this motion the Merton school was able to prove two important theorems:

Merton rule (or mean speed theorem): a body moving with a uniformly accelerated motion covers the same distance in a given time as a body moving for the same duration with a uniform speed equal to its mean (or average) speed.

Second theorem: The distance covered in the first half of a uniformly accelerated motion starting from rest is the same as a third of the distance covered in the second half.

Concept check

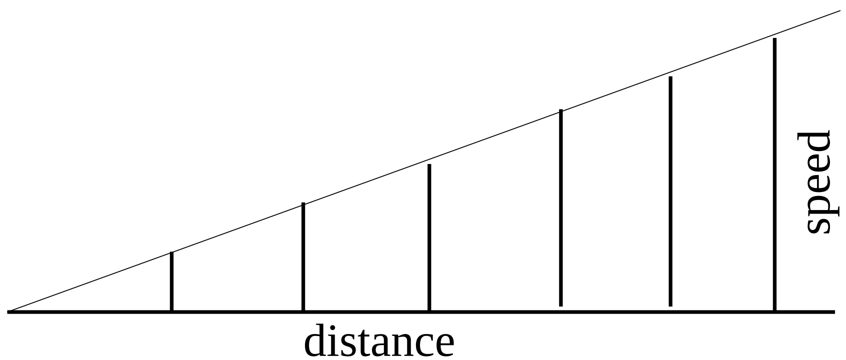
How would you go about proving these two Merton rules?

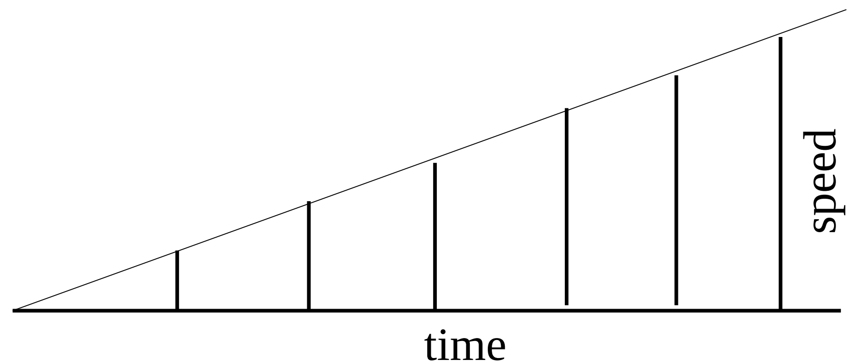
Are they actually correct?

If so, why, if not, why not?

Nicole Oresme's graphical representation (c.1360)

Oresme developed a graphical representation that can be extremely valuable to visualize motion (and many other things) and even facilitates mathematical proofs.

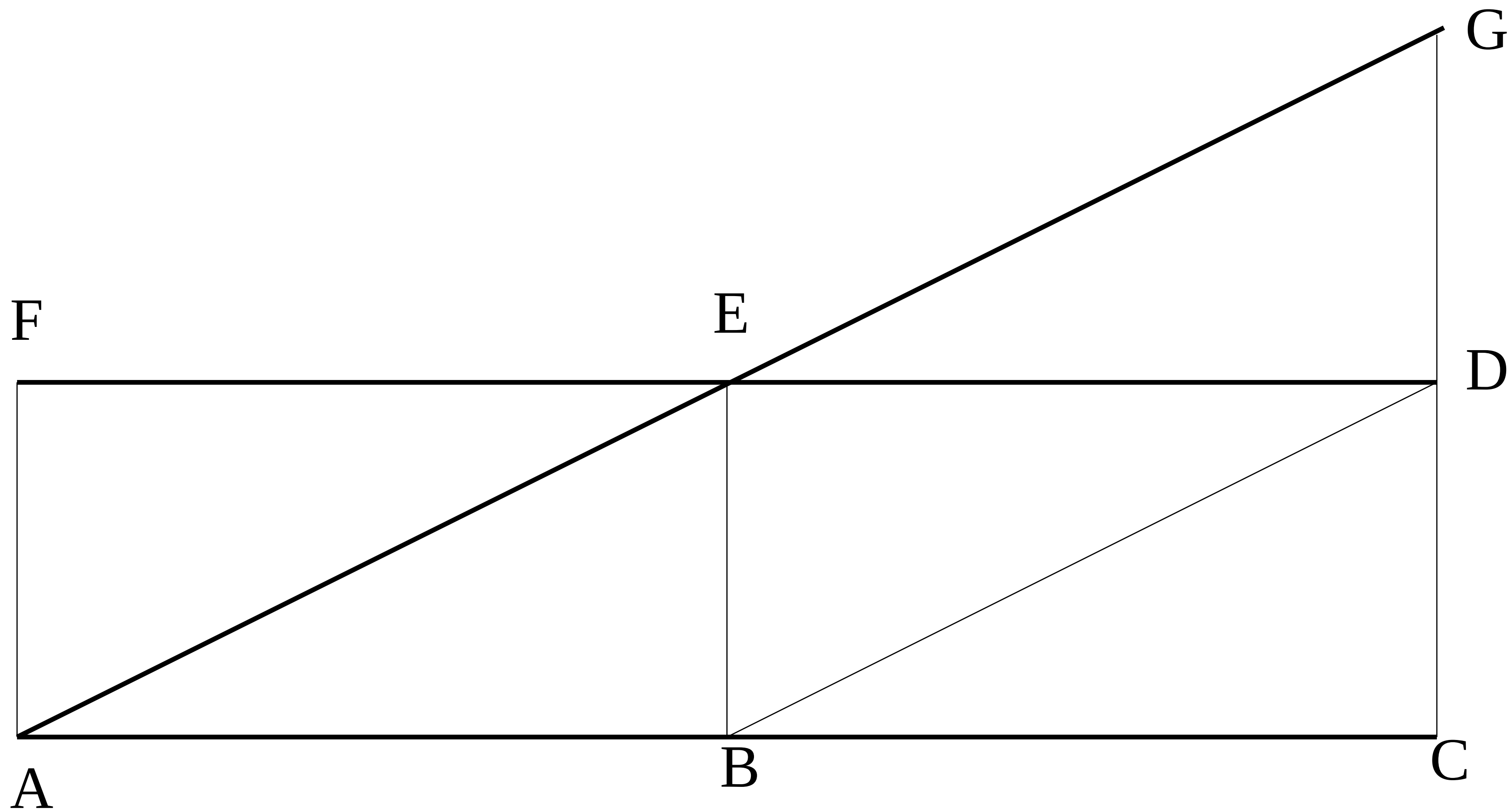




And examining these graphs Oresme realized that the distance traveled corresponds exactly to the area under the velocity/time graph!

How would that help you with proving the rules for uniformly accelerated motion the Merton group came up with?

Proof of the Merton Rules



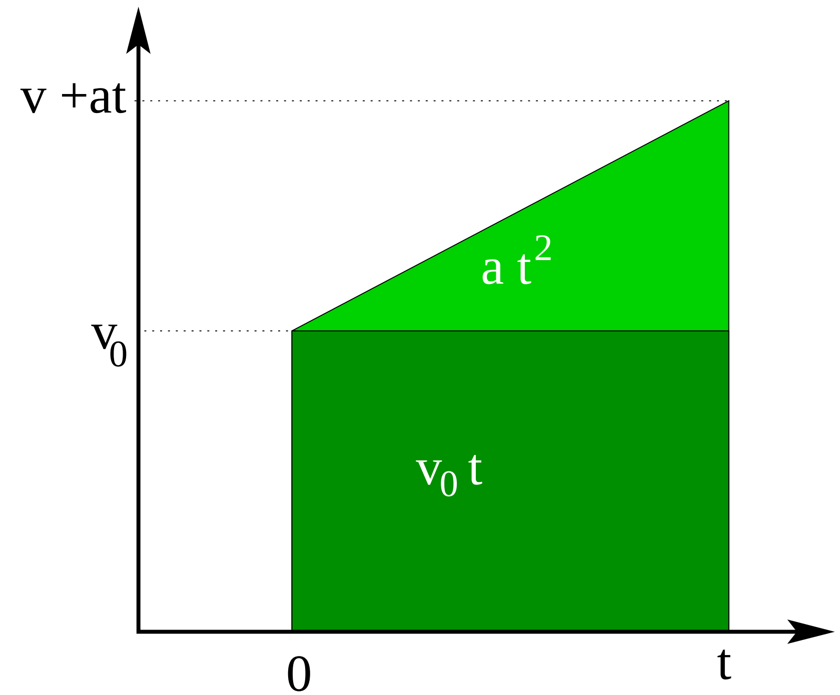
Proof of the Merton Rules

So how would you use to prove the merton rules using the graph on the previous page:

Merton rule 1: Distance travelled: area of triangle AGC, distance travelled by movement with average velocity: Area of rectangle AFDC. Counting the number of equal triangles, they have the same area, and therefore the same distance has been traversed!

Merton rule 2: Distance travelled during the second half of the motion is 3 times larger than the distance travelled in the first half: Area inside BEGC is 3 triangles and AEB is one triangle.

Modern version of the Merton rule



Medieval scientists often expressed their results in terms of ratios. Modern usage utilizes functions and algebraic expressions.
When we examine the v-t graph on the right we can read off the functional expressions we are interested in:

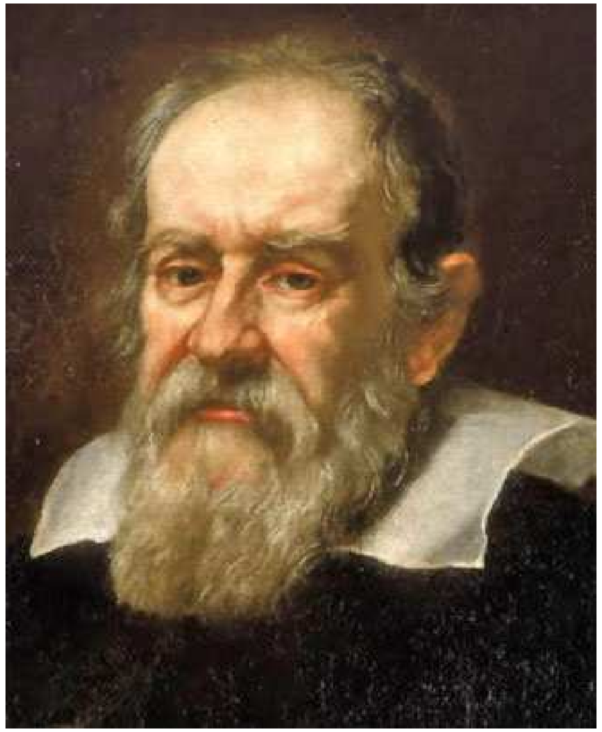
$$v(t) = v_0 + at \quad (1)$$

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2 \quad (2)$$

Remember that the distance traveled corresponds to the area under the curve. The velocity linearly increases from v_0 to $v_0 + at$, where a, the slope of the line, is the acceleration. The distance traveled to is then the original distance at $t = 0$ called x_0 plus the distance that would be traveled if there was no acceleration, shown as the dark green rectangle, plus the area of the light green triangle.

Galileo

Born: February 15, 1564, Pisa, Italy
Died: January 8, 1642, Arcetri, Italy
Education: University of Pisa
Galileo's law of falling: If airresistance is negligible then any two objects that are dropped together will fall together, regardless of their weight and their shapes, and regardless of the substance of which they are made.



Inclined planes

In class experiments

These experiments essentially allow you to slow down time so that you can *see* uniformly accelerated motion that is just too fast when you are examining freely falling objects!

Inclined planes - result: law of inertia

Law of inertia

A body that is subject to no external influences (also called external forces) will stay at rest if it was at rest to begin with and will keep moving if it was moving to begin with; in the latter case, its motion will be in a straight line at an unchanging speed. In other words, all bodies have inertia.

Free fall

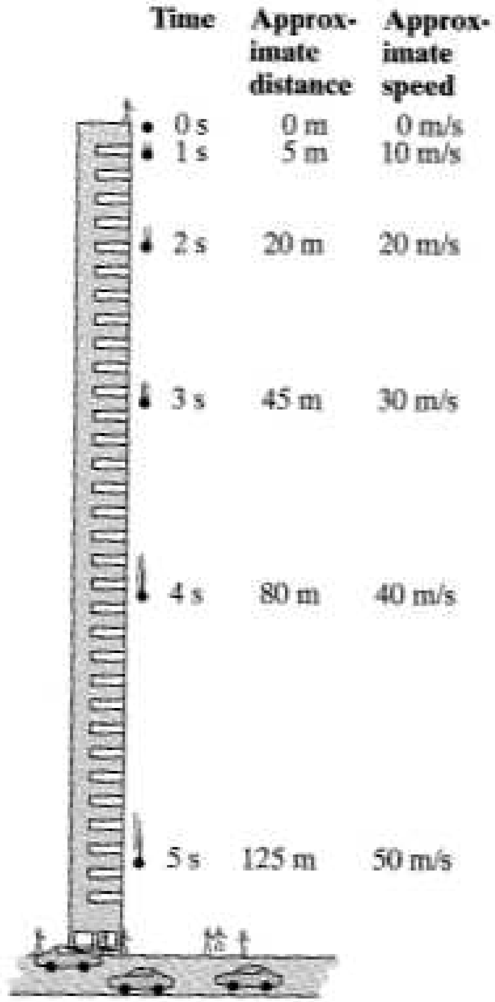


Figure 3.11
A freely falling ball dropped from the top of a tall building. The effect of air resistance is neglected in this illustration.

How does the speed change?

0s	1s	2s	3s	4s	5s
0m	10 m/s	20 m/s	30 m/s	40 m/s	50 m/s

Velocity increases in increments of 10 m/s.

Speed increases proportional to time: $s \propto t$

(This is **not** an equation).

How does the position change?

0s	1s	2s	3s	4s	5s
0m	5 m	20 m	45 m	80 m	125 m

Velocity increases in units of 5m.

Pattern : 0,1,4,9,16,25

Square numbers! $0^2, 1^2, 2^2, 3^2, 4^2, 5^2$

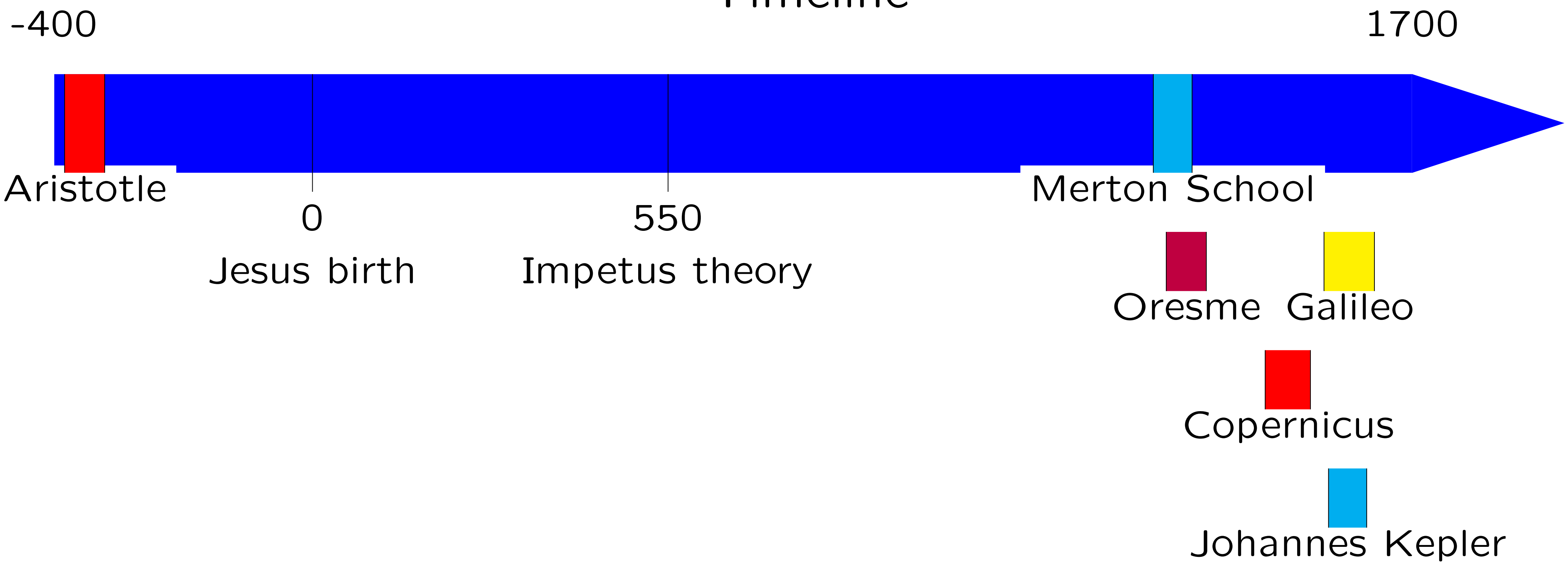
position increases as the square of time! $d \propto t^2$

Free fall – uniformly accelerated motion

The Merton school looked at uniformly accelerated motion purely as a methematical exercise. Noone knew if there was such a motion in reality.

Can you show from the experiments we looked at that free fall is a real example of uniformly accelerated motion?

Timeline


-400
1700
Aristotle
Merton School
0
550
Jesus birth
Impetus theory
Oresme
Galileo
Copernicus
Johannes Kepler

Summary

- Aristotelian physics
- Invention of mathematical kinematics
- Law of inertia
- velocity and acceleration
- free fall (constant acceleration)