Lecture 13

Fundamentals of Physics

Phys 120, Fall 2015

Special Relativity II

A. J. Wagner
North Dakota State University, Fargo, ND 58108

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Overview

- Review of special relativity so far
- Length depends on speed!
- Newton's equations don't work...
- Mass needs to depend on speed!
- $e = mc^2$

Review

1. Physical laws don't depend on (constant) speed.

2. The speed of light (in empty space) is always the same.

3. With a light clock we could see that time moves slower for fast moving objects

4. This lead to the twin paradox.

Have you had a chance to think this through?

Who fells that they understood the twin paradox?

a) I understood it
b) I sort of got it
c) This is still full of paradoxes

The twin paradox revisited

Consider this: it will appear to you on earth that when I leave earth with a high speed my time will progress more slowly.

However, as far as I am concerned you are moving away from me with a high speed, and **your** time is progressing more slowly.

So why is it that **I will be younger than you** on my return?

Simultaneous time

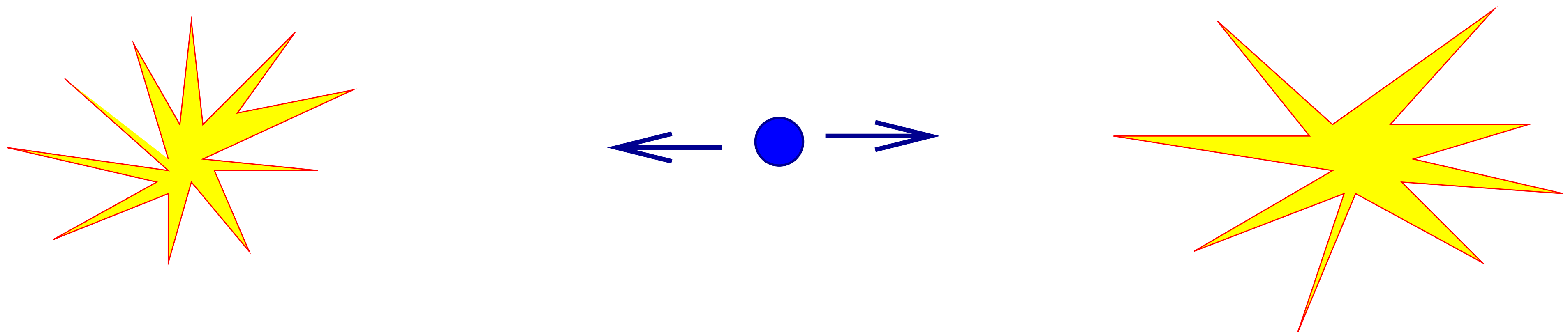
We can only claim that two events are simultaneous if they happen at exactly the same place. When we are at the same place we can synchronize watches, and there will be no disagreements.

However, once we are at different places you can no longer do that. The fastest way we can communicate is through light signals and that can establish a "before" and an "after". But for time differences that are less than can be communicated through light signals apparent simultaneity depends on your speed!

So for whom more time has passed is an undefined question while we are moving apart. But when **I turn around** I am no longer moving with a constant velocity and I can't use the argument that the same laws of Physics apply. (see General Relativity, next lecture).

When we reunite you remained at rest, so your observations of my time being slow remain correct, but when I turned around my experience of simultaneity changed.

Concept check



Two stars explode, and this is observed from earth. After calculating the distance to the star and taking the delay due to the finite speed of light into account scientists find that they exploded apparently at the same time.

At the same time an alien spaceship is passing earth with high velocity (0.99 c) and they also observe the explosions. Would the aliens agree that the explosions took place at the same time?

Explanation

Assume that the spaceship and the earth are at the same spot as the two explosions take place (according to an observer on earth). The stars are the same distance, but because the spaceship is moving away from the earth it will encounter the light from the star towards which it is moving sooner than the light of the star from which it is moving away. Therefore the aliens will conclude that the star towards which they are moving exploded first, and the star they are moving away from exploded only later.

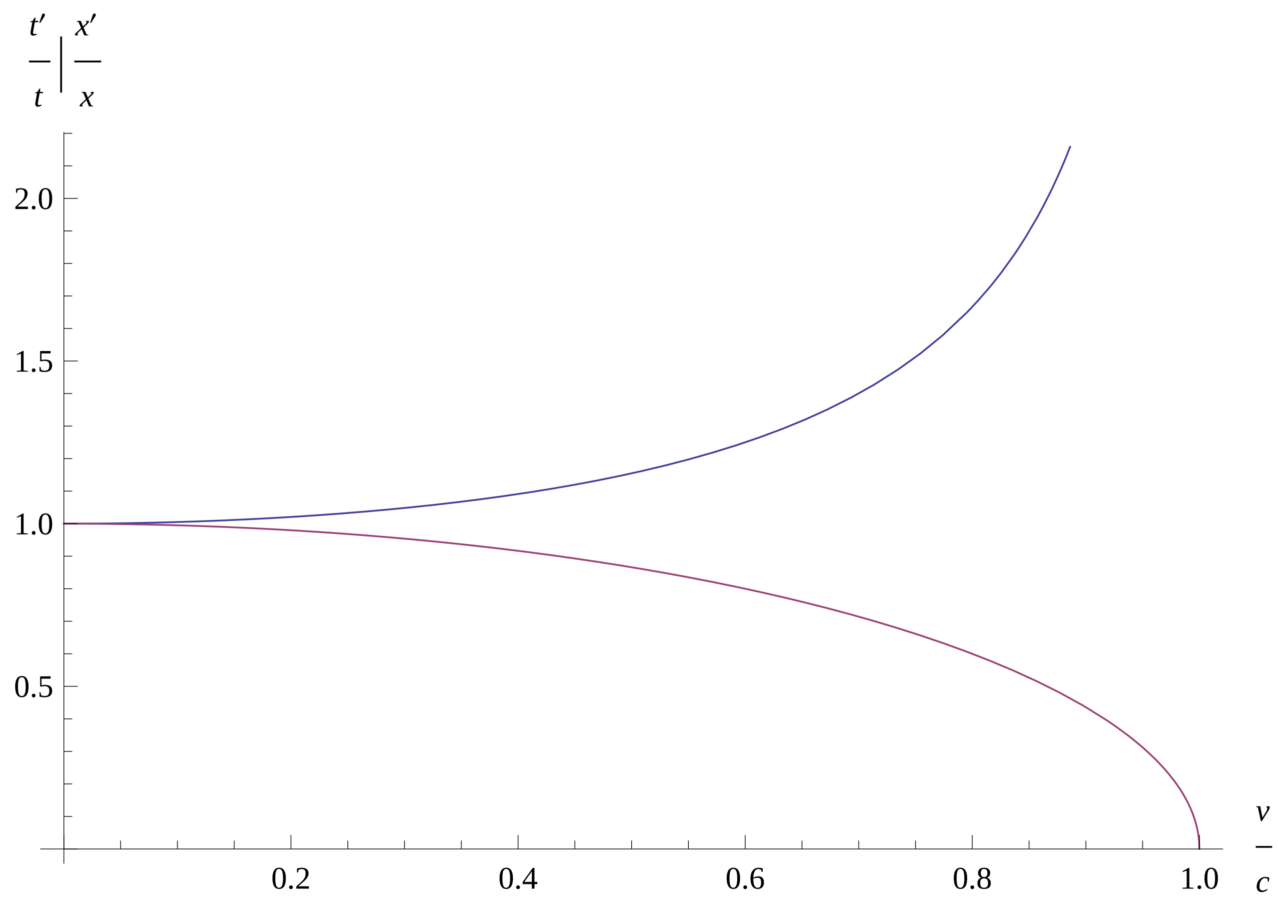
Length measurements

Let us consider how we measure a length. When we are moving with the same speed as the object we can just hold a yard-stick to the object, it will line up with the object for as long as we want and we can measure the length.

However, when we want to measure a moving object the yardstick will only be instantaneously lined up with the measuring stick. We have to know measure the position of the two ends of the object **at the same time**!

But what the same time is depends on the velocity, so time and space are related! What we find (through a similar argument than the clock argument) is that length get contracted at larger speeds.

Time expansion, length contraction



The numerical value for the space contraction is

$$x' = x\sqrt{1 - \frac{v^2}{c^2}}$$

Concept check

My spaceship which is 2m high and 20m long passes by earth at high speed. Is it possible for my spaceship to move past you fast enough so that it appears square?

a) Yes by moving about 0.1 c
b) Yes by moving about 0.99 c
c) No, because I would have to move at the speed of light to do this.
d) No, because objects do not change their shapes.

Concept check

You want to move, but you notice that your bed is too wide to pass through the door. Can you make it fit through, if you only move it fast enough?

a) No way. The bed will always remain too wide to fit through the door.
b) Yes, I have often observed that things will fit if you move them quickly
c) In principle yes, but only if you could move them **very** fast.

Newton's law

$$a = \frac{F}{m}$$

Newton's law tells us that if a constant force is acting on an object its velocity will continue to increase and it will increase even to more than the speed of light.

This means that an observer could catch up with a beam of light it sent out, so Newton's law is not consistent with the special theory of relativity!

If you push a one 1kg object that is at rest with a one Newton force you will accelerate it with 1 m/s^2. However, if the object is already moving, the special theory of relativity tells you that the acceleration is less!

(This effect is negligible for objects moving with speeds much smaller than c).

Mass

From your point of view the force applied to a faster object causes less acceleration. The property that describes the resistance to acceleration we called mass. So we need to introduce a velocity dependent mass!

Different observers will observe different masses, dependent on their relative velocity with respect to the object. Numerically the mass is given by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Classically mass is the same as amount of matter, but relativistically mass depends on velocity whereas the amount of matter remains the same.

How do we know that mass increases with speed?

If we have a charged particle we can measure its mass. Now if this particle is accelerated to speeds close to c and we try to divert it with an electric field then the deflection should be much less than classically predicted for a fast particle. This is actually observed in experiments!

Energy and mass

We just saw that when the kinetic energy of an object increases, so does its mass. Working from the theory of relativity and the law of conservation of energy, Einstein found that mass is connected to **every form of energy** in this fashion.

Einstein's analysis gives a formula that quantitatively relates the change in mass to the change in energy:

$$\text{change in mass} = \frac{\text{change in energy}}{\text{square of lightspeed}}$$

Concept check

If you stretch a rubber band, does its mass change?

a) Yes, because it gains potential energy
b) No, because the amount of matter remains the same

Answer

Assume that we stretch a strong rubber band with an average force of 300N over 0.6 m. The work is then W = Fd = 180 J.

So the mass is increased by

$$\text{change in mass} = \frac{180 \text{ J}}{(3 \times 10^8)^2} = 0.000000000000002 kg$$

Second Concept check

Consider two bar magnets that are touching.

If you pull them apart, what happens with the energy of the system?

Third example

Consider nuclear reactions. When a Uranium nucleus falls apart a large amount of energy is released. This means that the mass of the resulting atoms must be less than mass of the Uranium atom.

If 1 kg of uranium is fissioned the rest-mass loss is about 0.001 kg, whigh is a 0.1% mass decrease and easily detected. Experiments agree with Einstein's predictions.

Einstein's formula

Einstein took this a step further and he postulated that not only the change in mass is proportional to the change in energy, but that the mass itself is proportional to its energy:

$$m = \frac{E}{c^2}$$

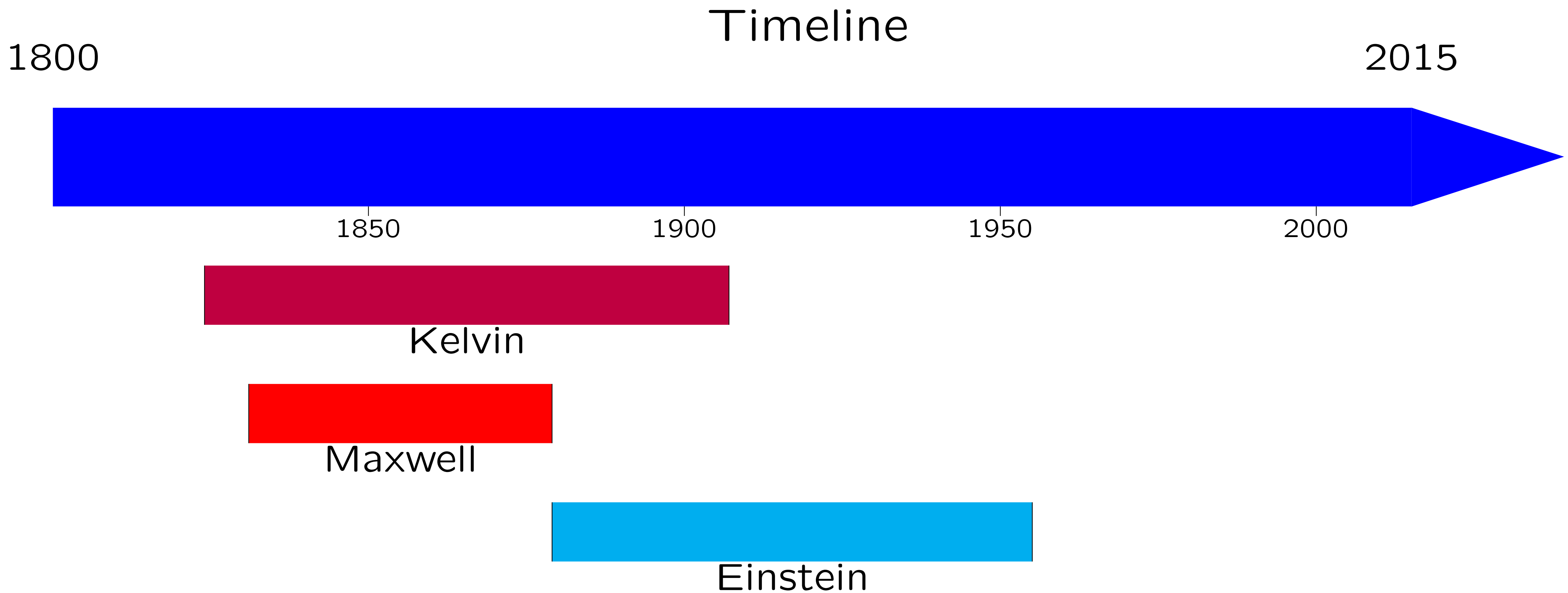
so all mass has energy and all energy has mass!

How do we know that $E = mc^2$?

This can be directly observed in **matter - antimatter anihilation** where all rest-mass vanishes and we are left with pure energy in the form of radiation.

The principle of mass energy Equivalence

$$\boxed{\text{Energy has mass; that is, energy has inertia. And mass has energy; that is mass has the ability to do work. The quantitative relation between energy of any system and the mass of that system is } E = mc^2.}$$


Timeline
1800
2015
1850
1900
1950
2000
Kelvin
Maxwell
Einstein

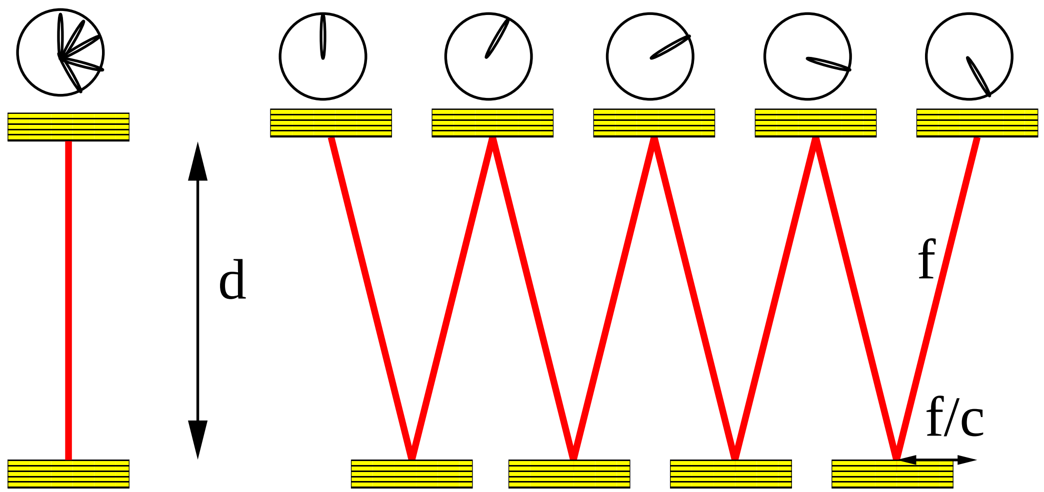
Summary

- time is not absolute, it depends on the speed of the observer. The faster the speed, the slower time progresses. Events that are simultaneous to one observer, are no simultaneous to another observer.

- space is not absolute. The length of an object depends on its speed! Faster objects are contracted.

- Energy and mass are related through $e = mc^2$, and in principle all energy has mass, and mass can be converted into energy.

Appendix: the time argument in formulas



The distance the light travels in the moving clock is f, so the horizontal distance has to be $vt = vf/c$. Pythagoras then gives:

$$d^2 + f^2 \frac{v^2}{c^2} = f^2 \qquad (1)$$

We can solve this for f to give

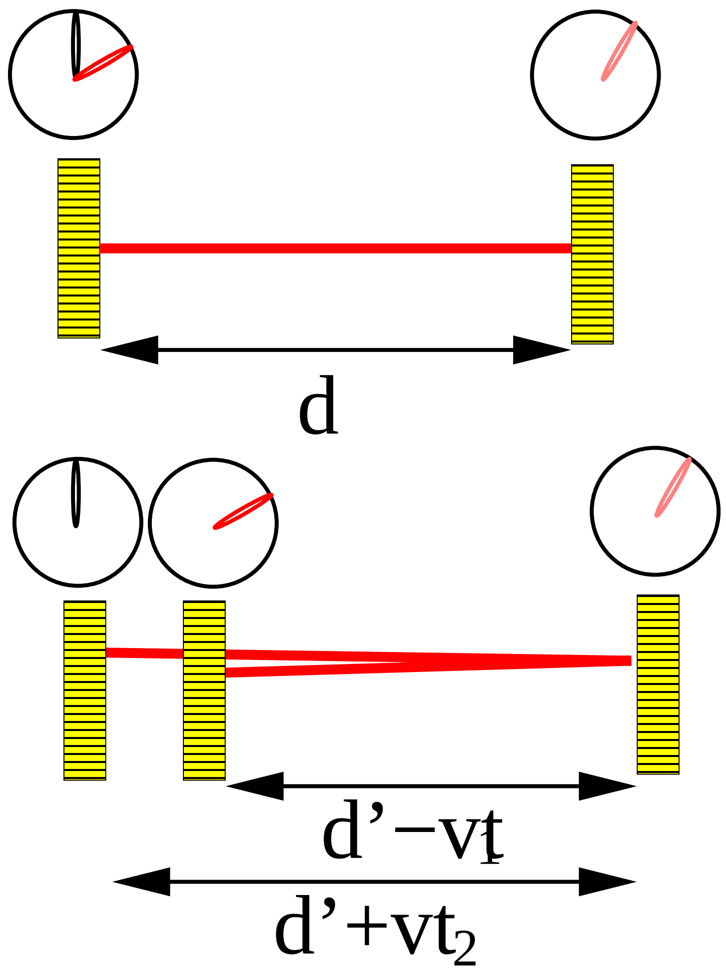
$$f = \frac{d}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (2)$$

and the time in the moving frame has to be $t' = f/c$ or

$$t' = \frac{f}{c} = \frac{d}{c} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (3)$$

where we used that $t = d/c$. This is exactly what I claimed in the last lecture.

Appendix: space contraction



For an observer at rest with respect to a ruler he can calculate the length of the ruler from the time it takes a light beam to return to its source from

$$2d' = c2t' \tag{4}$$

where t is the time it takes to travel from one mirror to the other.

For an observer on the ground, when the ruler is passing by with a velocity of v, the same expriment looks different. Since the ruler is moving the distance the light has to travel to hit the first mirror is $d + vt_1$ and the distance it has to travel on the return leg is reduced to $d - vt_2$. Here we have already allowed the size of the ruler d to be different for the stationary observer.

We have for the time it takes to reach the first mirror $t_1 = (d + vt_1)/c$ or

$$t_1 = \frac{d/c}{1 - \frac{v}{c}} \tag{5}$$

Similarly we get for t_2

$$t_2 = \frac{d/c}{1 + \frac{v}{c}} \tag{6}$$

and for the total roundtip time we get

$$t = t_1 + t_2 = \frac{2d/c}{1 - \frac{v^2}{c^2}} \tag{7}$$

We need to relate this to the length d' in the moving reference frame. We know that the time in the moving reference frame is dilate according to

$$t = \frac{t'}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{8}$$

and $t' = d'/c$. We can now get

$$d = d' \frac{1 - \frac{v^2}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} = d' \sqrt{1 - \frac{v^2}{c^2}} \tag{9}$$