MATH 165
FALL 2003
EXAM 1

1. (30 pt) Evaluate the following limits if they exist.

a) $\displaystyle\lim_{x\to 2}\frac{x^3-8}{x^3+x-10}$ b) $\displaystyle\lim_{x\to-\infty}\frac{\sqrt[3]{x^6-1}}{\sqrt{2x^4+3x}}$ c) $\displaystyle\lim_{x\to 0}x^2\tan^{-1}(\frac{1}{x})$

d) $\displaystyle\lim_{h\to 0}\frac{h}{\sqrt[4]{16+h}-2}$ e) $\displaystyle\lim_{\theta\to\pi}\frac{\sin(\theta)}{\theta}$

2. (30 pt) Find the derivative for each of the following functions.

a) $f(x)=x^6+2x+e^{2x}+2e^x+2$ b) $g(x)=x^2e^xF(x)$ c) $k(x)=\dfrac{e^x+x^2}{e^x-x}$

d) $h(x)=\dfrac{x^2e^x}{x^2-1}$ e) $T(x)=\dfrac{G(x)H(x)K(x)}{(G(x))^2+1}$

3. (15 pt) Use the definition of the derivative to find the derivative of the function $f(x)=e^{ax}$. You may use the fact that $\lim_{h\to 0}\frac{e^h-1}{h}=1$.

4. (8 pt) Suppose that the tangent line to the function $f(x)$ at $x=0$ is $y=2x+4$ and the tangent line to $g(x)$ at $x=0$ is $y=3x+1$. Find

$$\lim_{h\to 0}\frac{f(h)-4g(h)}{hg(h)}.$$

(Hint: Divide top and bottom by $g(h)$ and try to rework this as a quotient rule.)

5. (5 pt) We saw in class that the $\lim_{x\to 0}\sin(\frac{1}{x})$ does not exist. Give an appropriate value of ϵ (epsilon) that would justify this (and explain why your choice works).

6. (6 pt) Suppose that $f(x)$ is a one to one function that is differentiable everywhere. Suppose that the derivative of $f^{-1}(x)$ exists everywhere except at the point $(2,3)$. Find the tangent line to the function $f(x)$ at $(3,2)$.

7. (16 pt) Consider the following function.

$$f(x)=\begin{cases} ax^2+ax & \text{if } x<1 \\ -a^2x & \text{if } x\geq 1 \end{cases}$$

a) For what value(s) of a, if any, is $f(x)$ continous everywhere?
b) For what value(s) of a, if any, if $f(x)$ differentiable everywhere?