MATH 165
FALL 2004
EXAM 1

1. (36 pt) Evaluate the following limits if they exist.

a) $\lim\limits_{x\to 1} \dfrac{x^3-4x+3}{x^3+x^2-2}$ b) $\lim\limits_{x\to -\infty} \dfrac{\sqrt[3]{x^3+2x+2}}{4x+5}$ c) $\lim\limits_{x\to 0} f(\frac{1}{x})\sin(x)$, ($|f(x)| \leq 2$ for all x.)

d) $\lim\limits_{x\to -2} \tan(\dfrac{x+2}{x^2-4})$ e) $\lim\limits_{h\to 0} \dfrac{\sqrt[5]{2x+2h}-\sqrt[5]{2x}}{h}$ f) $\lim\limits_{x\to -\infty} (\sqrt{x^2+ax}-\sqrt{x^2+bx})$

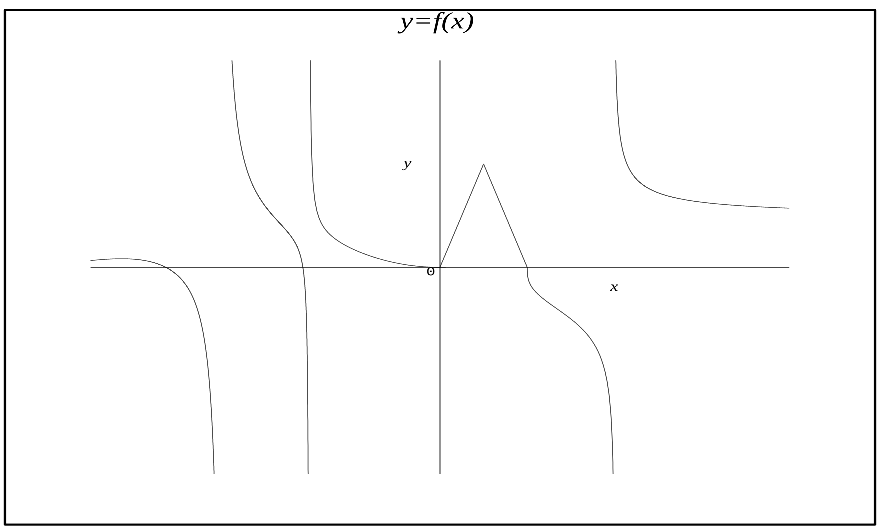
2. (21 pt) Find the derivative for each of the following functions (in part c, F, G, H and K are differentiable).

a) $f(x) = \dfrac{x^5-2x^3-x+1}{x}$ b) $g(x) = \sqrt[3]{x}e^{2x}$ c) $k(x) = \dfrac{e^x F(x)G(x)}{(H(x))^2-K(x)+1}$

3. (12 pt) Use the definition of the derivative to find the derivative of the function $f(x) = \sqrt{x^2+1}$.

4. (8 pt) Suppose that $f(x)$ is a differentiable function and let $F(x) = f(\frac{1}{x})$. Use the definition of the derivative to find $F'(x)$. (Hint: after you set this up, multiply the top and bottom by $\frac{1}{x+h} - \frac{1}{x}$.)

5. (11 pt) Sketch the pictured function in your exam book and use this to sketch a graph of its derivative.



6. (10 pt) Consider the function

$$f(x) = \begin{cases} ax+b & \text{if } x \geq 0, \\ bx^2+ax & \text{if } x < 0. \end{cases}$$

a) Find value(s) of a and b (if any) such that $f(x)$ is continuous everywhere (and explain your answer).
b) Find value(s) of a and b (if any) such that $f(x)$ is differentiable everywhere (and explain your answer).

7. (12 pt) Consider the function $f(x) = \begin{cases} |x|\sin(\frac{1}{x}) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$

a) Explain why $f(x)$ is continuous at 0.
b) Explain why $f(x)$ is continuous everywhere.
c) Explain why you cannot really "draw" an accurate picture of the graph of $y = f(x)$.