1. (36 pt) Evaluate the following limits if they exist.
   
   a) \( \lim_{x \to 1} \frac{x^3 - 4x + 3}{x^3 + x^2 - 2} \)
   
   b) \( \lim_{x \to -\infty} \frac{\sqrt{x^3 + 3x^2 + 2}}{4x + 5} \)
   
   c) \( \lim_{x \to 0} f(x) \sin(x), (|f(x)| \leq 2 \text{ for all } x.) \)
   
   d) \( \lim_{x \to -2} \tan\left(\frac{x + 2}{x^2 - 4}\right) \)
   
   e) \( \lim_{h \to 0} \frac{\sqrt{2x + 2h} - \sqrt{2x}}{h} \)
   
   f) \( \lim_{x \to -\infty} (\sqrt{x^2 + ax} - \sqrt{x^2 + bx}) \)

2. (21 pt) Find the derivative for each of the following functions (in part c, \( F, G, H \) and \( K \) are differentiable).
   
   a) \( f(x) = \frac{x^5 - 2x^3 - x + 1}{x} \)
   
   b) \( g(x) = \sqrt{x}e^{2x} \)
   
   c) \( k(x) = \frac{e^x F(x)G(x)}{(H(x))^2 - K(x) + 1} \)

3. (12 pt) Use the definition of the derivative to find the derivative of the function \( f(x) = \sqrt{x^2 + 1} \).

4. (8 pt) Suppose that \( f(x) \) is a differentiable function and let \( F(x) = f\left(\frac{1}{x}\right) \). Use the definition of the derivative to find \( F'(x) \). (Hint: after you set this up, multiply the top and bottom by \( \frac{1}{x^2 - 1} \).)

5. (11 pt) Sketch the pictured function in your exam book and use this to sketch a graph of its derivative.

6. (10 pt) Consider the function
   
   \[ f(x) = \begin{cases} 
   ax + b & \text{if } x \geq 0, \\
   bx^2 + ax & \text{if } x < 0.
   \end{cases} \]
   
   a) Find value(s) of \( a \) and \( b \) (if any) such that \( f(x) \) is continuous everywhere (and explain your answer).
   
   b) Find value(s) of \( a \) and \( b \) (if any) such that \( f(x) \) is differentiable everywhere (and explain your answer).

7. (12 pt) Consider the function \( f(x) = \begin{cases} 
   |x| \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0, \\
   0 & \text{if } x = 0.
   \end{cases} \)
   
   a) Explain why \( f(x) \) is continuous at 0.
   
   b) Explain why \( f(x) \) is continuous everywhere.
   
   c) Explain why you cannot really “draw” an accurate picture of the graph of \( y = f(x) \).