

MATH 165
FALL 2005
EXAM 1

1. (36 pt) Evaluate the following limits.

a) $\lim_{x \rightarrow 0} \frac{\sin(x^2)}{\tan(x) + 2}$ b) $\lim_{t \rightarrow 3} \frac{t^3 - 27}{t^2 - 5t + 6}$ c) $\lim_{x \rightarrow \infty} (\sqrt{x^2 + ax + b} - \sqrt{x^2 + cx + d})$
d) $\lim_{t \rightarrow -\infty} \frac{t}{\sqrt{t^2 + 1}}$ e) $\lim_{h \rightarrow 0} \frac{h}{\sqrt[3]{2a + 2h} - \sqrt[3]{2a}}$ f) $\lim_{x \rightarrow \infty} \frac{x \sin(x)}{x^4 + 1}$

2. (24 pt) Find the derivative of each of the following functions.

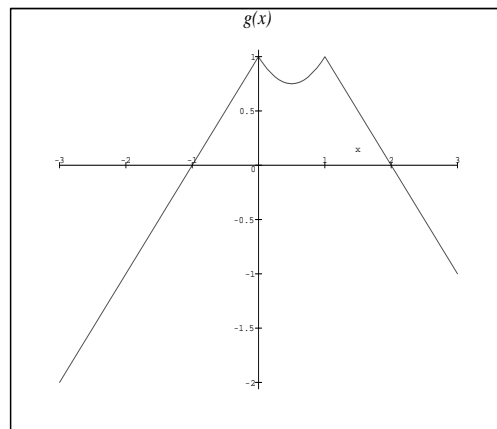
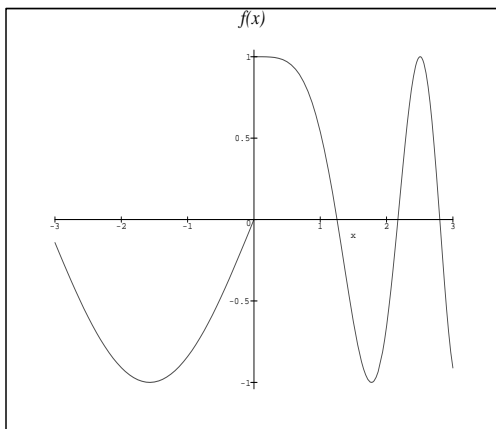
a) $f(x) = \frac{ae^{2x} + 1}{be^x + 1}$ b) $g(x) = (x^4 + 1)(x^2 + 1)(e^x + 2)$ c) $h(x) = \frac{F(x)G(x)H(x)}{\sin^2(x) + \cos^2(x) + 1}$
d) $k(x) = \frac{e^x \sqrt{x}}{x^3 + x^2 + \frac{1}{x}}$

3. (10 pt) Let f be a differentiable function. Find the derivative of the function $g(x) = \sqrt[3]{f(x)}$.

4. (12 pt) Use the definition of the derivative to compute the derivative of the following functions.

a) $f(x) = x|x|$.
b) $g(x) = F(ax)$, where F is a differentiable function.

5. (10 pt) Consider the functions pictured below.



- a) For what values of x is the function $f(g(x))$ continuous (and justify your answer)?
b) For what values of x is the function $g(f(x))$ continuous (and justify your answer)?

6. (10 pt) Consider the function

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0; \\ 0 & \text{if } x = 0. \end{cases}$$

- a) Explain why $f(x)$ is continuous everywhere.
b) Explain why $f(x)$ is differentiable at 0.

7. (8 pt) Find all (horizontal and vertical) asymptotes of the function $f(x) = e^{-\frac{1}{|x|}}$.