1. (36 pt) Evaluate the following limits.
   
   a) \( \lim_{x \to 3} \frac{x^2 - 9}{x^3 - 27} \)
   b) \( \lim_{x \to -\infty} \frac{1 - x}{\sqrt{x^2 + 3}} \)
   c) \( \lim_{x \to \infty} (\sqrt{a^2x^2 + bx} - (ax + c)), a > 0 \)
   d) \( \lim_{t \to -\infty} \frac{\sqrt{64t^9 + t^8 + 2}}{\sqrt{81t^6 + 43t^4 + 2}} \)
   e) \( \lim_{h \to 0} \frac{\sqrt{a + h} - \sqrt{a}}{h} \)
   f) \( \lim_{x \to 1} \ln(\tan(\frac{\sqrt{x} - 1}{x - 1})) \)

2. (28 pt) Find the derivative of each of the following functions.
   
   a) \( f(x) = \log_2(\tan(\tan^{-1}(2x))) \)
   b) \( g(x) = xe^{-x}F(x) \)
   c) \( h(x) = \frac{e^{2x}}{x^2 + G(x)} \)
   d) \( k(x) = \frac{s(x)}{x^5 + x^2 - 7}, \) where \( s'(x) = \sec(x) \)

3. (9 pt) We say that a function is increasing if \( x_1 < x_2 \) implies that \( f(x_1) < f(x_2) \) and decreasing if \( x_1 < x_2 \) implies that \( f(x_1) > f(x_2) \). Show that if \( f(x) \) is continuous and one to one on \( (-\infty, \infty) \), then \( f(x) \) is either increasing or decreasing (hint: use the intermediate value theorem).

4. (10 pt) Use the definition of the derivative to compute the derivative of the following functions.
   
   a) \( f(x) = e^{ax}, a \neq 0 \) (hint: \( \lim_{h \to 0} \frac{e^{ah} - 1}{h} = 1 \)).
   b) \( g(x) = \frac{ax}{bx+c} \).

5. (6 pt) Consider the function
   
   \[ f(x) = \begin{cases} 
   |x|, & \text{if } x \text{ is rational;} \\
   0, & \text{if } x \text{ is irrational.} 
   \end{cases} \]

   Show that \( \lim_{x \to 0} f(x) = 0 \).

6. (15 pt) Suppose that \( f(x) \) is continuous and one to one on \( (-\infty, \infty) \).
   
   a) Explain why \( F(x) = e^{f(x)} \) is one to one and find \( F^{-1}(x) \).
   b) Explain why \( F(x) \) must have at least one horizontal asymptote (hint: problem 3).
   c) Can \( F(x) \) have any vertical asymptotes? Why or why not?

7. (6 pt) Consider the function \( f(x) = \sin^2(\tan^{-1}(x)) \).
   
   a) \( f(x) = \frac{x^2}{x^2 + 1} \).
   b) \( \frac{d}{dx}(\sin^2(\tan^{-1}(x))) \).