MATH 165
FALL 2006
EXAM 1

1. (36 pt) Evaluate the following limits.

a) $\lim_{x\to 3} \frac{x^2-9}{x^3-27}$ b) $\lim_{x\to -\infty} \frac{1-x}{\sqrt{x^2+3}}$ c) $\lim_{x\to\infty} (\sqrt{a^2x^2+bx}-(ax+c)), a>0$

d) $\lim_{t\to\infty} \frac{\sqrt[6]{64t^9+t^8+2}}{\sqrt[4]{81t^6+43t^5+2}}$ e) $\lim_{h\to 0} \frac{\sqrt[45]{a+h}-\sqrt[45]{a}}{h}$ f) $\lim_{x\to 1} \ln(\tan(\frac{\sqrt{x}-1}{x-1}))$

2. (28 pt) Find the derivative of each of the following functions.

a) $f(x) = \log_2(\tan(\tan^{-1}(2^x)))$ b) $g(x) = xe^{-x}F(x)$ c) $h(x) = \frac{\frac{e^{2x}}{x+1}}{\frac{1}{x^2}+G(x)}$

d) $k(x) = \frac{s(x)}{x^5+x^2-7}$, where $s'(x) = \sec(x)$

3. (9 pt) We say that a function is increasing if $x_1 < x_2$ implies that $f(x_1) < f(x_2)$ and decreasing if $x_1 < x_2$ implies that $f(x_1) > f(x_2)$. Show that if $f(x)$ is continuous and one to one on $(-\infty, \infty)$, then $f(x)$ is either increasing or decreasing (hint: use the intermediate value theorem).

4. (10 pt) Use the definition of the derivative to compute the derivative of the following functions.
a) $f(x) = e^{ax}, a \neq 0$ (hint: $\lim_{h\to 0} \frac{e^h-1}{h} = 1$).
b) $g(x) = \frac{ax}{bx+c}$.

5. (6 pt) Consider the function

$$f(x) = \begin{cases} |x|, & \text{if } x \text{ is rational;} \\ 0, & \text{if } x \text{ is irrational.} \end{cases}$$

Show that $\lim_{x\to 0} f(x) = 0$.

6. (15 pt) Suppose that $f(x)$ is continuous and one to one on $(-\infty, \infty)$.
a) Explain why $F(x) = e^{f(x)}$ is one to one and find $F^{-1}(x)$.
b) Explain why $F(x)$ must have at least one horizontal asymptote (hint: problem 3).
c) Can $F(x)$ have any vertical asymptotes? Why or why not?

7. (6 pt) Consider the function $f(x) = \sin^2(\tan^{-1}(x))$.
a) Show that $f(x) = \frac{x^2}{x^2+1}$.
b) Find $\frac{d}{dx}(\sin^2(\tan^{-1}(x)))$.