MATH 165
FALL 2007
EXAM 1

1. (36 pt) Evaluate the following limits.

a) $\lim_{x\to 2} \frac{\sqrt{x^2+5}+x-5}{x-2}$ b) $\lim_{x\to 0} \frac{\tan(ax+1)}{\sin(b^2x+1)}$ c) $\lim_{x\to -\infty} (\frac{3x\sqrt{x^2+1}}{2x^2+x})$

d) $\lim_{t\to 1} \sqrt{\frac{t^3-2t+1}{t^4-1}}$ e) $\lim_{h\to 0} \frac{\tan(a^2+2ah+h^2)-\tan(a^2)}{h}$ f) $\lim_{x\to 0} x\sin(g(\frac{1}{x}))$

2. (24 pt) Find the derivative of each of the following functions.

a) $f(x) = \frac{\sin(2x)}{2\tan(x)+1}$ b) $g(x) = e^{x^2}\sin(ax)\cos(bx)$ c) $h(x) = \frac{\sin(\sin(\sin(e^{\sin(x)})))}{x^2+1}$

d) $k(x) = \sqrt{1+\sqrt[3]{2+\sqrt[4]{xe^x}}}$

3. Let $f(x) \leq g(x) \leq h(x)$ be functions.

a) (5 pt) Write the $\delta - \epsilon$ definition of the meaning of the statement "$\lim_{x\to a} g(x) = L$".
b) (3 pt) Show use the $\delta - \epsilon$ definition to show that if $\lim_{x\to a} f(x) = L = \lim_{x\to a} h(x)$, then $\lim_{x\to a} g(x) = L$.

4. (12 pt) Use the definition of the derivative to compute the derivative of the following functions.

a) $f(x) = \frac{2x}{x+1}$.
b) $g(x) = F(ax+b)$, where F is a differentiable function.

5. (8 pt) Find all possible tangent lines to the curve $f(x) = x^3+6x^2+8$ that go through the point $(0,0)$.

6. (10 pt) Consider the function $f(x) = x^{\frac{1}{3}}$.

a) Where is this function differentiable? Geometrically explain why the derivative does not exist at the origin.
b) Suppose that the derivative of $g(x)$ does not exist at $x = a$. Does it follow that the derivative of $f(g(x))$ also does not exist at $x = a$?

7. (12 pt) A more realistic model of a falling ball (incorporating air resistance) might be

$$s(t) = v_T(t + \frac{v_T}{g}e^{-\frac{gt}{v_T}}) - \frac{v_T^2}{g}$$

where t is time, $s(t)$ is distance fallen, g is the constant acceleration due to gravity, and v_T is a constant called the terminal velocity.

a) Find the velocity, $v(t)$, of the falling ball and evaluate $\lim_{t\to\infty} v(t)$.
b) Find the acceleration, $a(t)$, of the falling fall and evaluate $\lim_{t\to\infty} a(t)$.