1. (30 pt) Evaluate the following limits.

a) $\lim\limits_{x\to 1} \dfrac{bx^3 - b}{ax^2 - a}$

b) $\lim\limits_{x\to -\infty} \dfrac{\sqrt{a^2 x^2 - 1}}{\sqrt[4]{b^4 x^4 + 4x - b}}$

c) $\lim\limits_{x\to 0} \dfrac{\tan^2(ax)}{bx\sin(cx)}$

d) $\lim\limits_{t\to\infty} \sec(\dfrac{\pi t}{4t + 2})$

e) $\lim\limits_{h\to 0} \dfrac{e^{(x+h)^2} - e^{x^2}}{h}$

f) $\lim\limits_{x\to 2} (x^2 - 4x + 4)\sin(\dfrac{x+1}{x-2})$

2. (32 pt) Find the derivative of each of the following functions.

a) $f(x) = \dfrac{x}{e^x \sin(x) + 1}$

b) $g(x) = \tan(x\cos(x)e^{x^2})$

c) $h(x) = e^{\sin(e^{\sin(x)})}$

d) $k(x) = \sqrt{x^2 + \sqrt[3]{x\tan(x) + e^{5x}}}$

3. (8 pt) Show that any line of nonzero slope must intersect the graph of $f(x) = \sin(x)$ at least once.

4. (10 pt) Use the definition of the derivative to compute the derivative of the following functions.
   a) $f(x) = \sin(3x)$.
   b) $g(x) = F(x^2)$, where F is a differentiable function.

5. (10 pt) Let $a, b > 1$ both be integers. How many horizontal tangents does the function

$$f(x) = \dfrac{x^a}{x^b + 1}$$

possess?

6. (15 pt) Consider the function

$$f(x) = \begin{cases} x^2 \sin(\frac{1}{x}), & \text{if } x \neq 0; \\ 0, & \text{if } x = 0. \end{cases}$$

   a) Find $f'(0)$ if it exists.
   b) Find a formula for $f'(x)$.
   c) Is $f'(x)$ continuous?

7. (5 pt) Use the precise definition of the limit to show that if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = \infty$ then $\lim_{x\to a}(f(x) + g(x)) = \infty$.