MATH 165
FALL 2009
EXAM 1

1. (36 pt) Evaluate the following limits.

a) $\lim_{x\to\infty} \frac{f(x)\sin(x)}{x^2+1}$, where $x \le f(x) \le 4x$ if $x > 0$. b) $\lim_{\theta\to 0} \frac{\theta^2 - \sin^2(2\theta)}{\tan^2(3\theta)}$

c) $\lim_{x\to-\infty} \frac{\sqrt[3]{2x^3+3x+1}}{4x-5}$ d) $\lim_{h\to 0} \frac{(f(2a+2h))^2 - 2f(2a+2h)f(2a) + (f(2a))^2}{h^2}$

e) $\lim_{x\to 2\pi} \frac{\sin(x)}{x}$ f) $\lim_{x\to 0} \frac{e^x-1}{e^{2x}-1}$

2. (24 pt) Find the derivative of each of the following functions.

a) $f(x) = x^2 e^{2x}\sin(3x)$ b) $g(x) = \frac{xe^x}{x^2 - \sec(x) + 1}$

c) $h(x) = \sin(x + \sin(x^2 + e^{(x^2\tan(x) - \sin(x^3))}))$ d) $k(x) = f(f(x^2))g(\sin(x))$

3. (10 pt) Use the definition of the derivative to find the derivative of the function $f(x) = \frac{1}{\sqrt{4x+5}}$.

4. (10 pt) Consider the function

$$f(x) = \begin{cases} x^2, & \text{if } x \text{ is rational;} \\ 0, & \text{if } x \text{ is irrational.} \end{cases}$$

a) Find all points at which $f(x)$ is continuous (you may use the result of problem 7 if you wish).
b) Find all points at which $f(x)$ is differentiable.

5. (16 pt) One model for a falling object encountering air resistance is given by

$$s(t) = \frac{gm}{k}t - \frac{c_1 m}{k}e^{-\frac{k}{m}t} + c_2,$$

where g, m, k, c_1, c_2 are all positive constants and $s(t)$ denotes the distance fallen after t seconds.
a) Find the velocity, $v(t)$, of the object.
b) Find the acceleration, $a(t)$, of the object.
c) Find $\lim_{t\to\infty} v(t)$.
d) Find $\lim_{t\to\infty} a(t)$.

6. (9 pt) Consider the function $f(x) = x^2$ and let (a, b) be a point in the plane. Find conditions on a and b such that:
a) There are no tangent lines to $f(x) = x^2$ that pass through (a, b).
b) There is precisely one tangent line to $f(x) = x^2$ that passes through (a, b).
c) There are two tangent lines to $f(x) = x^2$ that pass through (a, b).

7. (5 pt) State the precise definition of $\lim_{x\to a} f(x) = L$ and give an appropriate value of ϵ to demonstrate that $\lim_{x\to a} f(x)$ does not exist (where $a \ne 0$ and $f(x)$ is the function from problem 4).