MATH 165
FALL 2010
EXAM 1

1. (36 pt) Evaluate the following limits.

a) $\lim_{x\to 2} \sin\left(\sqrt[3]{\frac{x^2-4}{x^3-x^2-4}}\right)$ b) $\lim_{\theta\to 0} \frac{\tan(\sin(\theta))}{\sin(5\theta)}$ c) $\lim_{x\to\infty} \frac{\lfloor ax+b\rfloor}{x}$

d) $\lim_{h\to 0} \frac{\sqrt[5]{32+h}-2}{h}$ e) $\lim_{x\to 0} \frac{|ax+b|-|ax-b|}{x}$ f) $\lim_{x\to-\infty} \frac{3x}{\sqrt{4x^2+9}}$

2. (24 pt) Find the derivative of each of the following functions.

a) $f(x) = \sin(x\sin(x\sin(x)))$ b) $g(x) = F(\sin(x))G(e^{x^2})H(\tan(x^2\sin(2x)))$

c) $h(x) = \frac{ax^2+bx+c}{Ax+B}$ d) $k(x) = \sqrt{x - \sqrt[3]{\sin(xe^x+x^4) + \sqrt[4]{e^{x^2+x}+\tan(2x+1)}}}$

3. (10 pt) Use the definition of the derivative to find the derivative of the function $f(x) = g(ax+b)$ where $g(x)$ is a differentiable function.

4. (10 pt) Consider the functions

$$f(x) = \begin{cases} x+1, \text{ if } x \geq 0; \\ -x^2, \text{ if } x < 0. \end{cases}$$

and

$$g(x) = b^2x^2 + bx + b^2.$$

Find all values of b such that $g(f(x))$ is continuous everywhere.

5. (16 pt) An object moves according to the position function $s(t) = \cos(3t) + 9t + 2$ (where $s(t)$ is in feet and t is in seconds).

a) Find a function that gives the velocity of the object at any time t.
b) What are the maximum and minimum values of the velocity?
c) Find a function that gives the acceleration of the object.
d) Find the smallest $t > 0$ such that the acceleration at time t is 0.

6. Consider the function $f(x) = \frac{x^2}{x+1}$.

a) (5 pt) Find the tangent line to $f(x)$ at the point $(a, \frac{a^2}{a+1})$.
b) (3 pt) What happens to the tangent line from the previous part as $a \longrightarrow \infty$?

7. (6 pt) State the precise definition of $\lim_{x\to a} f(x) = L$ and use this definition to show that $\lim_{x\to 0} |x| = 0$.