MATH 165
FALL 2011
EXAM 1

1. (30 pt) Evaluate the following limits:

a) $\lim_{x\to 3} \dfrac{x^2-9}{x^3-2x^2-9}$ b) $\lim_{t\to -\infty} \dfrac{\sqrt{4t^4+t^2+3}}{t^2+6}$ c) $\lim_{x\to 1} \dfrac{\sqrt{x}-1}{\sqrt[3]{x}-1}$

d) $\lim_{h\to 0} \dfrac{\tan(\frac{\pi}{4}+h)-1}{h}$ e) $\lim_{x\to\infty} (\sqrt{x^2+Ax+B}-(x+a))$ f) $\lim_{x\to 2} |2-x| \tan^{-1}(\dfrac{x}{x-2})$

2. (32 pt) Find the derivative for the following functions.

a) $f(x) = \tan(e^{\sin(x)}) \sin(\tan(x))$ b) $g(x) = \sin(e^{\sin(e^{\sin(e^x)})})$

c) $h(x) = f(x)g(x)f(g(x))g(f(x))$ d) $k(x) = \dfrac{x\cos(5x^2)e^{x^3}}{\sqrt{x^2+x+1}}$

3. (10 pt) Consider the function

$$f(x) = \begin{cases} x, \text{ if } x \text{ is rational;} \\ x^2, \text{ if } x \text{ is irrational.} \end{cases}$$

Use the precise definition to show that $\lim_{x\to 0} f(x) = 0$.

4. (10 pt) Use the definition of the derivative to find the derivative of the function $f(x) = x - \lfloor x \rfloor$.

5. (10 pt) An assassin shoots his target and drops his gun from a tall building. A construction zone radar sign clocks the speed of the gun at 160 feet per second when the gun hits the pavement. If each floor is 10 feet tall, from which floor did the assassin fire? (You may assume that the rifle falls according to the law $s(t) = 16t^2$.)

6. (10 pt) Consider the function given by

$$f(x) = \begin{cases} 2x+1, \text{ if } x > 0; \\ -x^2+3, \text{ if } x \leq 0. \end{cases}$$

If $g(x)$ is continous and one to one, is it possible for $g(f(x))$ to be continuous everywhere? Why or why not?

7. (8 pt) Show that at this moment there are two places on opposite sides of the earth where the temperature is exactly the same.