MATH 165
FALL 2012
EXAM 1

1. (30 pt) Evaluate the following limits:

a) $\lim_{x\to-\infty}(x+\sqrt{x^2+ax})$ b) $\lim_{x\to-1}\frac{x^2+x}{x^3+2x+3}$ c) $\lim_{h\to0}\frac{\sqrt[5]{x+h}-\sqrt[5]{x}}{h}$

d) $\lim_{x\to-\infty}\frac{3x+2}{\sqrt[6]{64x^6+x^2+1}}$ e) $\lim_{x\to1}|x-1|f(\frac{3}{x^2-1})$, where $-9<f(x)<5$.

f) $\lim_{x\to2}\sin(\tan^{-1}(e^{\frac{x^2-4}{x-2}}))$

2. (32 pt) Find the derivative for the following functions.

a) $f(x)=\sin(\sqrt{x+\sqrt{x+\sqrt{x^3\tan(x^2)}}})$ b) $g(x)=\frac{x\sin(x^6e^{x^3})}{\cos(x)\tan(2x)+x^4}$ c) $h(x)=e^xe^{e^x}e^{e^{e^x}}$

d) $k(x)=f(g(f(e^{\sin(x)})))g(f(g(x^2)))$

3. (8 pt) Consider the function

$$f(x)=\sqrt[3]{x}\cos^2(x^4).$$

Use the precise definition of the limit to show that $\lim_{x\to0}f(x)=0$.

4. (10 pt) Let $f(x)$ be a differentiable function. Use the definition of the derivative to find the derivative of the function $g(x)=f(x^2)$.

5. (12 pt) Consider the function $f(x)=3x^{\frac{1}{3}}-x$.
   a) Where is $f(x)$ continuous?
   b) Where is $f(x)$ differentiable?
   c) Where is $f'(x)$ positive and where is it negative?
   d) Use this information to sketch the graph of $f(x)$.

6. (12 pt) Let $a,b>0$ and consider the function given by

$$f(x)=\begin{cases}\frac{|ax|-|bx|}{x}, & \text{if } x\neq0,\\ c, & \text{if } x=0.\end{cases}$$

   a) What conditions are needed on a,b and c to ensure that $f(x)$ is continuous everywhere?
   b) What conditions are needed on a,b and c to ensure that $f(x)$ is differentiable everywhere?
   c) What is the $\lim_{x\to-\infty}f(x)$?

7. (6 pt) Let $f(x)$ be a continuous function on $[0,1]$ such that $0\leq f(x)\leq1$. Show that there is a c in $[0,1]$ such that $f(c)=c$.