MATH 165
FALL 2003
EXAM 2

1. (36 pt) Evaluate the following derivatives:

a) $f(x) = \sin(\sqrt[3]{x^2+1})$ b) $g(x) = \sqrt{\dfrac{2x\tan^3(x)}{x^3-1}}$ c) $h(x) = \sin(\cos(\tan(e^{2x})))$

d) $k(x) = \sqrt{x + \sqrt{x + \sqrt{\sin(x^2)}}}$ e) $m(x) = \tan^{-1}(x^{x^x})$

f) $F(t) = \ln(|\cos(at)|)$

2. (20 pt) Find the maximum and minimum values of the following functions on the specified intervals

a) $f(x) = x^{\frac{5}{3}} - 5x^{\frac{2}{3}}$ on $[-1, 5]$.

b) $g(x) = a\sin(x) + \cos(x)$ on $[0, \frac{\pi}{2}]$, $a \geq 0$.

3. (12 pt) A (spherical) star is observed and we wish to calculate its surface area. Suppose that our observed radius has a relative error of .01 (1% error). Use differentials to estimate the relative error in using our measured radius to compute the surface area of the star. Will our estimate of the error be an overestimate or an underestimate?

4. (10 pt) Consider the curve defined by the equations $y^2 + x^3 = 3xy$. Find all points where the slope of the tangent line is 0. You may ignore the point (0,0).

5. (12 pt) A cup of water in the shape of a cylinder of base radius r is set out in the sun. The water begins to evaporate and the rate of evaporation is proportional to the exposed surface area. Find rate of decrease of the depth of the water and answer this question: "if you want to have a glass of water and have the water take the longest time to evaporate should you pick a short and fat glass or a tall and thin one?"

6. (12 pt) Sand is being dumped into a pile at a constant rate, and the coarseness of the sand is such that the pile is always half as high as it is wide. If the height of the pile is growing at 1 foot per minute when the pile is 5 feet high, then how fast is sand being dumped onto the pile?

7. (8 pt) Let $g(x)$ be a differentiable function such that $g'(x)$ is never 0. Show that the function

$$f(x) = \tan^{-1}(g(x))$$

has at most one real root.