MATH 165
FALL 2004
EXAM 2

1. (40 pt) Evaluate the following derivatives:

a) $f(x) = \sin^3((2x)^{\ln(3x)})$ b) $g(x) = \dfrac{\sqrt[3]{2x+3}\sqrt[5]{3x+7}(x^2+3x)^7}{\cos^4(x^2)(x^3+1)^2\sqrt[3]{(1-4x)^5}}$ c) $h(x) = e^{e^{e^x}}$

d) $k(x) = \sin^{-1}(\sin(2x)\cos(x) + \cos(2x)\sin(x))$ e) $m(x) = \sqrt[4]{x + \sqrt[3]{\sin(x) + \sqrt{e^{x^2} + 3x}}}$

2. (15 pt) Find the maximum and minimum values of $f(x) = x^{\frac{1}{3}} + (2-x)^{\frac{1}{3}}$ on the interval $[-1, 2]$.

3. (10 pt) A large spherical star of radius R is absorbing the mass (which we will approximate by volume) off of a smaller star of radius r. Suppose that this process allows both objects to remain perfect spheres. Show that

$$\frac{dR}{dt} = -\frac{a}{S}\left(\frac{dr}{dt}\right)$$

where S is the surface area of the large star and a is the surface area of the smaller one.

4. (10 pt) Consider a function defined implicitly by $x^y = y^x$ (here you may assume that both x and y are positive). Find the tangent line at the point (a, a) ($a > 0$, $a \neq e$). Also find the slope of the tangent lines at $(2, 4)$ and $(4, 2)$. (Extra credit...can be turned in anytime before the final exam: Show that if (a, b) is a point on the curve where the slope of the tangent line is 1, then $a = b$.)

5. (15 pt) A silo is built by putting a cone of height and radius 8 feet on top of a (circular) cylinder of height 20 feet and radius 8 feet. An ice storm hits and covers the roof of the silo with ice about 2 inches thick and the cylindrical side with ice about $\frac{1}{2}$ of an inch thick. Use differentials to estimate the volume of the ice on the silo.

6. (12 pt) (*The Tidal Wave*) Suppose that the area under one wave of a sine curve is given by $A = \frac{2}{\pi}hD$ where h is the height (amplitude) of the wave and D is the width of the base of the wave. Suppose that the area under this curve remains constant and the base width is decreasing at a rate of 100 ft/sec when the height of the wave is twice the base width. How fast is the height of the wave increasing at this instant?

7. (8 pt) Show that if $0 < |a| < |b|$ then the equation $bx = \cos(ax)$ has at most one real solution.