MATH 165
FALL 2005
EXAM 2

1. (40 pt) Evaluate the following derivatives:

a) $f(x) = \sin(e^{\sin(x^2)})$ b) $g(x) = F(x)^{H(x)^{K(x)}}$ c) $h(x) = \dfrac{\sqrt{2x+3}\sqrt[3]{5x^2-4}\sin^2(3x)}{(\ln(x))^3(x^2+3x+3)^7}$

d) $k(x) = \ln(\ln(\dfrac{xe^x}{x^2+1}))$ e) $b(x) = \tan^{-1}(\sqrt{\sec^2(x)-1})$

2. (9 pt) Find the tangent line to $xy^2 = \sin(x+y)$ at the point $(0, \pi)$.

3. (12 pt) Find the maximum and minimum values of the function $f(x) = \cos(2x) + 2\sin(x)$ on the interval $[0, \pi]$.

4. (15 pt) A pool is constructed by putting an inverted spherical cap into the ground (the volume of such an object is given by $V = \pi D^2(R - \frac{1}{3}D)$ where D is the height of the cap and R is the radius of the sphere from which it is obtained). You have a hose filling the pool at a constant rate. You notice that the depth of the water is changing at 1 inch per second when the depth of the water is 1 foot. If the radius of the sphere from which the pool is constructed is 3 feet, how fast is water entering the pool (in cubic feet per second)?

5. (12 pt) Two runners run clockwise on a circular track. Both runners run at constant (although perhaps different) speeds. One runner is in a lane that is 30 feet from the central point and does five laps per minute. The second runner is 60 feet from the central point and does 3 laps per minute. How fast is the distance between the runners changing (in feet per second) when the second runner is at the 12 o'clock position and the first runner is at the 4 o'clock position?

6. (6 pt) Let $f(x) = \tan^{-1}(x) + \cot^{-1}(x)$. Find $f(\pi\sqrt{2})$ and explain how you got your answer.

7. (10 pt) You wish to paint the outside of a large sphere of radius 20 feet with three coats of paint. If a single coat of paint is $\frac{1}{10}$ of an inch thick, use differentials to estimate the amount of cubic feet of paint that will be needed. Is your estimate of the amount of paint an underestimate or an overestimate?

8. (6 pt) Let $f(x)$ be a differentiable function such that $f'(x)$ is never 0. Show that there is at most one real number a such that $f(a) = e^{-f(a)}$.