# MATH 165
# FALL 2006
# EXAM 2

1. (42 pt) For the following functions, find the derivative.

   a) $f(x) = \sin(e^{\tan(x)})$ b) $g(x) = \dfrac{x^2 \sin^3(x)}{\sqrt[3]{x^2 + 2x}(e^{x^2} + 1)^{\frac{3}{7}}}$ c) $h(x) = \ln(\ln(1 + \cos(x)\ln(x)))$

   d) $k(x) = x^{\frac{\cos(x)}{\ln(x)}}$ e) $b(x) = \sqrt{e^x + \sqrt{x^x + \sqrt{x^3 + 1}}}$ f) $m(x) = \sin^{-1}(e^{x^2})$

2. (15 pt) Find the maximum and minimum values of $f(x) = x^{\frac{3}{5}}(8 + 3x)$ on $[-2, 2]$.

3. (10 pt) Consider the function $f(x) = x^5 + x$.
   a) Use implicit differentiation to find a formula for the derivative of $f^{-1}(x)$.
   b) Find the tangent line to $f^{-1}(x)$ at the point (2,1).

4. (15 pt) The energy of an object is given by

$$E = mgh + \frac{1}{2}m(\frac{dh}{dt})^2$$

where $m$ is the (constant) mass of the object $h$ is its height above the ground, and $g$ is a constant. If the energy of the system is increasing at 50 foot pounds per second at the instant when the object's velocity is 10 feet per second, what is the acceleration (in terms of $m$ and $g$)?

5. (10 pt) A spinning sphere has rotational kinetic energy given by:

$$E = \frac{1}{5}mR^2\omega^2$$

where $m$ is the (constant) mass of the sphere, $R$ is the radius of the sphere, and $\omega$ is its angular speed (measured in radians per second). If the energy remains constant, use differentials to estimate the change in angular speed that corresponds to a 1% decrease in the radius of the sphere.

6. (10 pt) On a sunny day, a 60 foot tall flagpole begins to topple over smoothly so that the angle of the flagpole with the ground decreases at $\frac{\pi}{6}$ radians per second. Suppose that the sun is in the sky at an angle such that all shadows cast form an angle of $\frac{\pi}{3}$ radians with the ground. How fast is the tip of the shadow cast by the falling flagpole moving 1 second before the flagpole hits the ground?

7. (8 pt) Show that if $a > k > 0$ then the equation

$$ax + b = \tan^{-1}(kx)$$

has at most one real solution.