MATH 165
FALL 2007
EXAM 2

1. (40 pt) Evaluate the following derivatives:

a) $f(x) = \sin(e^x + \tan(\sqrt{x^3 - 2x}))$ b) $g(x) = \dfrac{\sqrt{x^2+1}\sqrt[3]{x^3+x}\,\ln(\sin(2x))}{\sin^2(x^2)(x^4 - x^2 + 1)^8}$ c) $h(x) = e^{x^{x^2}}$

d) $k(x) = \tan^{-1}(x^{\ln(x)})$

2. (16 pt) Find the maximum and minimum values of the following functions on the given intervals.
a) $f(x) = \frac{\sin(ax)}{\cos(ax)-2}$ on $[0, \frac{\pi}{a}]$, $a > 0$.
b) $g(x) = x^{\frac{1}{3}}(4-x)^{\frac{1}{3}}$ on $[-1, 4]$.

3. (12 pt) The sun rises at 6am and sets at 6pm (and goes directly overhead). You are standing in the shadow of a mountain and notice that the shadow of the mountain is decreasing at 2 feet per second at 10am. How high is the mountain?

4. (16 pt) Two people start walking from the same point at 12pm. The first person walks due south at 5 miles per hour and the second walks thirty degrees north of due east at 4 miles per hour. How fast is the distance between them changing at 2pm?

5. (8 pt) The volume of a sphere is given by $V = \frac{4}{3}\pi R^3$ and the surface area is given by $S = 4\pi R^2$ where R is the radius of the sphere. Find a formula for the volume of the sphere in terms of the surface area. Use this formula and differentials to answer the following. Estimate how small you need the relative error of the surface area so that the relative error in the calculated volume of the sphere is no more $\frac{1}{100}$.

6. (8 pt) Find the tangent line at the origin to the function defined implicitly by

$$e^{ax+by} = \cos(Ax + By)$$

where a, b, A, B are constants with $b \neq 0$.

7. (10 pt) Let $f(x)$ be a positive ($f(x) > 0$), differentiable function such that $f'(x)$ is either always positive or always negative.
a) Show that $G(x) = f(x)^{f(x)}$ has at most one local extreme value.
b) Show that if $G(x)$ does have an extreme value at $(c, G(c))$, it must be a local minimum.