1. (40 pt) The following define functions of \( y = f(x) \) explicitly or implicitly. In all cases, find \( y' \).

   a) \( f(x) = (x \sin(x))^{\tan(x)} \)
   b) \( xy + y^3 = x^5 + y \)
   c) \( f(x) = \frac{x^2}{\text{sin}((2x)^2) + 3} \)
   d) \( x^y = y^x \)
   e) \( f(x) = \tan^{-1}(\frac{x^3(x + 1)^{\frac{1}{2}}(x^2 + 6)^{21}}{\text{sin}^3(x^2)\sqrt{x^2 + 1}}) \)

2. (20 pt) Find the maximum and minimum values of \( f(x) = x^\frac{5}{3} + 20x^{-\frac{1}{3}} \) on the interval \([1, 8] \).

3. (12 pt) At night you are standing 20 feet from a very tall building. A car with headlights 3 feet off the ground is coming toward you at a constant speed. If you are 6 feet tall and your shadow on the tall building is growing at a rate of 10 feet per second when it is 12 feet tall, how long do you have to get out of the way of the approaching car?

4. (12 pt) A rocket takes off at time \( t = 0 \) and a TV camera \( a \) feet away from the blast-off point stays focused on the rocket as it ascends. If the rocket rises at the constant speed \( v \), how fast must the camera increase its angle to stay focused on the rocket (in terms of \( v, a \) and the height of the rocket only)? What happens to your answer as (height of the rocket) \( \rightarrow \infty \)?

5. (10 pt) You measure the side of a cube and find that it is of length \( s \). Use differentials to estimate the maximum relative error in your measurement of the side if you want:

   a) The calculated volume of the cube to have a relative error of no more than \( \frac{1}{100} \).
   b) The calculated surface area of the cube to have a relative error of no more than \( \frac{1}{100} \).

6. (8 pt) Let \( f(x) \) and \( g(x) \) be continuous functions that are nonzero on \([a, b] \) and differentiable on \((a, b) \). Suppose further that \( f(a) = f(b) \) and \( g(a) = g(b) \). Show that there is a number \( c \) in \((a, b) \) such that

   \[
   \frac{f'(c)}{f(c)} = \frac{g'(c)}{g(c)}. 
   \]

7. (8 pt) Suppose that a sample of some radioactive element has a half-life of \( T \) years. How long (in terms of \( T \)) will it take given sample to decay to 1% of its original radioactive mass?