MATH 165
FALL 2008
EXAM 2

1. (40 pt) The following define functions of $y = f(x)$ explicitly or implicitly. In all cases, find y'.

a) $f(x) = (x\sin(x))^{\tan(x)}$ b) $xy + y^3 = x^5 + y$ c) $f(x) = \dfrac{x^{x^2}}{\sin((2x)^x) + 3}$

d) $x^y = y^x$ e) $f(x) = \tan^{-1}\left(\dfrac{x^3(x+1)^{\frac{1}{2}}(x^2+6)^{21}}{\sin^3(x^2)\sqrt{x^2+1}}\right)$

2. (20 pt) Find the maximum and minimum values of $f(x) = x^{\frac{5}{3}} + 20x^{-\frac{1}{3}}$ on the interval $[1, 8]$.

3. (12 pt) At night you are standing 20 feet from a very tall building. A car with headlights 3 feet off the ground is coming toward you at a constant speed. If you are 6 feet tall and your shadow on the tall building is growing at a rate of 10 feet per second when it is 12 feet tall, how long do you have to get out of the way of the approaching car?

4. (12 pt) A rocket takes off at time $t = 0$ and a TV camera a feet away from the blast-off point stays focused on the rocket as it ascends. If the rocket rises at the constant speed v, how fast must the camera increase its angle to stay focused on the rocket (in terms of v, a and the height of the rocket only)? What happens to your answer as (height of the rocket) $\longrightarrow \infty$?

5. (10 pt) You measure the side of a cube and find that it is of length s. Use differentials to estimate the maximum relative error in your measurement of the side if you want:

a) The calculated volume of the cube to have a relative error of no more than $\frac{1}{100}$.
b) The calculated surface area of the cube to have a relative error of no more than $\frac{1}{100}$.

6. (8 pt) Let $f(x)$ and $g(x)$ be continuous functions that are nonzero on $[a, b]$ and differentiable on (a, b). Suppose further that $f(a) = f(b)$ and $g(a) = g(b)$. Show that there is a number c in (a, b) such that

$$\frac{f'(c)}{f(c)} = \frac{g'(c)}{g(c)}.$$

7. (8 pt) Suppose that a sample of some radioactive element has a half-life of T years. How long (in terms of T) will it take given sample to decay to 1% of its original radioactive mass?