

MATH 165
FALL 2009
EXAM 2

1. (40 pt) The following define functions of $y = f(x)$ explicitly or implicitly. In all cases, find y' .
 - a) $f(x) = g(h(x)^{\sin(x)})$
 - b) $f(x) = \ln(\sin(x^2))$
 - c) $f(x) = \tan(\sin(x^{\sin(\ln(x)+xe^x)}))$
 - d) $f(x) = x^{\frac{1}{\ln(x)}}$
 - e) $x^x = y^y$

2. (20 pt) Find the maximum and minimum values of the following functions.
 - a) $f(x) = \cos(x) - \cos(2x)$ on the interval $[0, \frac{\pi}{2}]$.
 - b) $f(x) = \sqrt[3]{x}(x^2 - 4)$ on the interval $[-2, 3]$.

3. (12 pt) A ball is dropped from 50 feet high, 30 feet from a 50 foot high street light. How fast is the shadow of the ball moving along the ground $\frac{1}{2}$ second later (you may assume that the ball falls obeying the formula $s = 16t^2$).

4. (12 pt) Suppose a raindrop is (and remains) perfectly spherical and picks up extra water in a misty cloud at a rate that is proportional to its surface area. Show that the radius of the raindrop increases at a constant rate.

5. (10 pt) The roof of a tall tower is in the shape of a cone with the height and radius equal (both 10 feet). An ice storm hits and coats the roof with a uniformly thick coating of ice that is a inches thick.
 - a) Use differentials to estimate the volume of ice that is coating the roof. Is your estimate an underestimate or an overestimate (and why)?
 - b) Is the roof can withstand 6250 lbs before collapse and water weigh 62.5 lbs per cubic foot, estimate the smallest value of a (thickness of coating of the ice) that will cause the roof to fail.

6. (8 pt) A piece of cloth is recovered by some archaeologists. Upon examination, it is found that 75% of the original Carbon 14 remains in the cloth. If the half-life of Carbon 14 is 5730 years, estimate the age of the piece of cloth.

7. (8 pt) Suppose that $f(0) = 1$ and $f'(x) = 2$ for all values of x . Use the Mean Value Theorem to show that $f(x) = 2x + 1$