MATH 165
FALL 2010
EXAM 2

1. (40 pt) For the following functions of x (defined explicitly or implicitly), find y'. If implicitly defined, you need not solve for y'.

a) $F(x) = g(x^{\frac{1}{x}})$ b) $G(x) = x^{(2x)^{\sin(3x)}}$ c) $H(x) = \tan^{-1}(x^x + \sin(x)^{\sin(x)})$

d) $K(x) = \dfrac{\sqrt{x+2}\sqrt[3]{2x+3}\sqrt[4]{3x+4}}{\sin^2(x)e^{x^2}\ln(\tan(x)}$ e) $x^3 + y^2 + xy = y^{2x+3y}$

2. (20 pt) Find the maximum and minimum values of the following functions.
a) $f(x) = \frac{x}{x^2+9}$ on the interval $[-1, 10]$.
b) $g(x) = \tan^{-1}(2x) - x$ on the interval $[0, \pi]$.

3. (12 pt) A flaming asteroid is flying at a constant height and constant velocity v. It passes A feet over a suspended horizontal platform of length L that is B feet above the ground.
a) How fast is the shadow caused by the asteroid growing when the asteroid is D feet beyond the leftmost point of the platform?
b) How fast is the distance between the point below the asteroid and the rightmost point of the shadow growing at that instant?

4. (12 pt) The hour hand of a clock is 3 inches and the minute hand is 6 inches. How fast is the distance between the tip of the hour hand and the minute hand changing at 2pm (in inches per minute)?

5. I want to paint a hemispherical dome of radius 50 feet with a coat of paint that is $\frac{1}{10}$ inches think, and I have to buy the paint.
a) (7 pt) Use differentials to estimate the amount of paint (in cubic feet) that I will need for this project.
b) (3 pt) If I buy exactly the amount of paint from part a), will I run out of paint? Explain.

6. (8 pt) Newton's Law of Cooling say that the change in temperture ($T(t)$) of an object is proportional to the difference between the object and the (constant) temperture of its surroundings (T_0).
a) Write an equation for Newton's Law of Cooling that reflects the above description.
b) Find the solution to this equation. You may freely use that the solution to the equation $\frac{dy}{dt} = ky$ is given by $y(t) = y(0)e^{kt}$.

7. (8 pt) Two runners run a marathon. They start at the same time and finish in an exact tie. Show that at some time during the race, the runners must have had exactly the same speed.