MATH 165
FALL 2011
EXAM 2

1. (32 pt) For the following functions of x (defined explicitly or implicitly), find y'. If implicitly defined, you need not solve for y'.

a) $F(x) = x^{x^2 \tan(x^2)} \sin(x)$ b) $G(x) = \sin(x)^{\cos(2x)^{\tan(3x)}}$ c) $\sin(y^{e^x}) = x^{y^x}$

d) $K(x) = \ln(\dfrac{\sqrt{x+2}\sqrt[3]{2x+3}\sqrt[4]{3x+4}}{\sin^2(x)e^{x^2}\ln(\tan(x))})$

2. (32 pt) Find the limits

a) $\lim\limits_{x\to a} \dfrac{x^{\frac{1}{n}} - a^{\frac{1}{n}}}{x^n - a^n}$ b) $\lim\limits_{x\to 0^+} (1 + \dfrac{|a|}{x})^{bx}$ c) $\lim\limits_{x\to\infty} \dfrac{x\cos(2x)}{x^2+1}$

d) $\lim\limits_{x\to 0} \dfrac{\sin(x^2) - x^2}{x^6}$

3. (10 pt) Find the maximum and minimum values of the function $f(x) = \ln(x^4 - x^2 + 1)$ on $[-\frac{1}{2}, 2]$

4. (10 pt) (*The minute hand is off*) Suppose that you have a clock and you know that the hour hand is keeping correct time and the clock itself gives the correct time at 2pm. If the minute hand of the clock is 12 inches long and the hour hand is 6 inches long and the distance between the tips of the hour hand and the minute hand is changing at $-\frac{\pi}{6}$ inches per minute at 2pm, what time will the clock read at 3pm?

5. (6 pt) Use differentials or a linear approximation to estimate $\sqrt[3]{25}$. Is your approximation too large or too small? Explain.

6. (10 pt) Two cars start at the same point and travel down two different straight roads that are at angle θ relative to each other. Show that if the two cars travel at a constant speed (say v and w) then the distance between them is changing at a constant rate.

7. (10 pt) Sketch the graph of $f(x) = 3x^{\frac{1}{3}} - x$.