MATH 165
FALL 2012
EXAM 2

1. (32 pt) The following define functions of $y = f(x)$ explicitly or implicitly. In all cases, find y'.

a) $f(x) = g((\sin(x))^x)h(x^{\sin(x)})$ b) $f(x) = \dfrac{(x+4)^5(3x+2)^9\sin^3(2x)}{\tan^2(x^2)\ln\ln(x)}$ c) $f(x) = x^{(2x)^{(3x)^{(4x)}}}$

d) $\sin(x^2y) = ye^{x^2}$

2. (24 pt) Find the following limits.

a) $\displaystyle\lim_{x\to 0}\frac{\sin(x^3)-x^3}{x^9}$ b) $\displaystyle\lim_{x\to 1}\frac{x^4+x^2-2}{x^5+2x-3}$ c) $\displaystyle\lim_{x\to\infty}\frac{3x^2+x\sin(4x)}{x^2+1}$

d) $\displaystyle\lim_{x\to\infty}(1+\frac{a}{x})^x$

3. (16 pt) Find the maximum and minimum values of the following functions.

a) $f(x) = 2\sin(ax)+\sin(2ax)$ on the interval $[0, \frac{\pi}{a}]$, $a > 0$.

b) $f(x) = x^{\frac{2}{3}}(2ax-5b)$ on the interval $[0,1]$ (here $0 < \frac{5}{2}b < a$).

4. (10 pt) A very tall crane of height h has a pulley and a light on top. The pulley is raising a girder of length L feet. The shadow is growing at a rate of $\frac{1}{5}L$ feet per second when the length of the shadow of the girder is $2L$ feet.

a) How high is the girder at this instant?

b) How fast is the girder rising at this instant?

5. (10 pt) A new roof is put on top of a flat-topped building. The long roof (of length 900 feet) has equal cross sections in the shape of an inverted parabola of height h (measured from the original roof to the vertex). The volume of this parabolic roof is given by $V = 12000h^2$ where h is in feet. If the roof is covered by a dusting of $\frac{1}{10}$ inches of snow and h is 30 feet, use differentials to estimate the weight of the snow on the roof (snow typically weighs about 15 lbs per cubic foot). Is the actual weight heavier or lighter than your estimate? Explain.

6. (10 pt) Sketch the graph of the function $f(x) = x^{\frac{2}{3}}(1-x^2)^{\frac{2}{3}}$. For your convenience, the first two derivatives of $f(x)$ are $f'(x) = \frac{2(3x^2-1)}{3x^{\frac{1}{3}}(x^2-1)^{\frac{1}{3}}}$ and $f''(x) = \frac{2(9x^4-12x^2-1)}{9x^{\frac{4}{3}}(x^2-1)^{\frac{4}{3}}}$. (The real roots of $9x^4-12x^2-1$ are approximately 1.19 and -1.19.)

7. (8 pt) Show that the function $f(x) = x^5+x^3+x+1$ has precisely one real root.