**MATH 165**
**FALL 2003**
**EXAM 3**

1. (24 pt) Evaluate the following limits.

a) $\lim_{t\to\infty} \dfrac{\cos(t) - 4t}{t+1}$ b) $\lim_{x\to 0} \dfrac{\cos(x^2) - 1}{3x^4}$ c) $\lim_{x\to\infty} (1 + \dfrac{a}{x})^{bx}$

d) $\lim_{x\to 0} \dfrac{(f(x))^2}{x\sin(2x)}$, where $f(0) = 0$ and $f'(0) = 7$.

2. (20 pt) Graph the function

$$f(x) = \frac{x}{\ln(|x|)}.$$

The first two derivatives of this function are

$$f'(x) = \frac{\ln(|x|) - 1}{(\ln(|x|))^2} \text{ and } f''(x) = \frac{2 - \ln(|x|)}{x(\ln(|x|))^3}.$$

3. (18 pt) Graph the function

$$g(x) = \frac{8 - x^3}{2x^2} = -\frac{1}{2}x + \frac{4}{x^2}$$

the first two derivatives are

$$g'(x) = \frac{-16 - x^3}{2x^3} \text{ and } g''(x) = \frac{24}{x^4}.$$

4. (18 pt) A 50 foot wide billboard sits forty feet from (and perpedicular to) a long straight road. Find the distance from the point on the road closest to the billboard where your view is best (that is, maximize the angle of your vision of the billboard).

5. (20 pt) A window is to be constructed by taking a rectangular pane of glass and putting equilateral triangular panes of glass on the top and bottom. If the perimeter of the window is to be $P$ feet, find the length and width of the rectangular pane of glass that will maximize the area of the window.

6. (10 pt) Suppose that you wish to find a root of the equation $f(x) = 0$ and you use Newton's method with first approximation $x_1$. Show that the $x_2$ that you obtain is the same as the result that you would get using differentials to approximate the root.