MATH 165
FALL 2004
EXAM 3

1. (32 pt) Evaluate the following limits:

   a) $\lim_{x\to 0} \dfrac{\tan^2(ax)}{\tan(bx^2)}, b \neq 0.$ b) $\lim_{x\to 0}(1+f(x))^{\frac{c}{x}}$, where $f(0)=0$ and $f'(x)$ is continuous at 0.

   c) $\lim_{x\to -\infty} \dfrac{\sqrt[3]{x^7+x^3-1}}{\sqrt[6]{2x^{14}-3x-1}}$ d) $\lim_{t\to\infty} \dfrac{\sin(2t)-2t}{t}$

2. (18 pt) Sketch the graph of $f(x)=\tan^{-1}(\ln(|x|))$. The first two derivatives of this function are

$$f'(x)=\frac{1}{x((\ln(|x|))^2+1)}$$

and

$$f''(x)=\frac{-(\ln(|x|)+1)^2}{x^2((\ln(|x|))^2+1)^2}.$$

3. (15 pt) Sketch the graph of $g(x)=\frac{x^3-2x+1}{x^2-1}=x-\frac{x-1}{x^2-1}$. The first two derivatives of this function are

$$g'(x)=\frac{(x-1)^2(x^2+2x+2)}{(x^2-1)^2}$$

and

$$g''(x)=\frac{-2(x-1)^3}{(x^2-1)^3}.$$

4. (18 pt) You make a trapezoid with three line segments of length a (and a fourth line segment, parallel to the opposite side, of any length that you need). What is the maximum area of such a trapezoid?

5. (15 pt) A propane tank is to be made of a cylinder of radius R and length h surmounted by two hemispheres each of radius R (one hemisphere on each end). Suppose the volume of this tank is to be the constant value V. Find the value of h that minimizes the surface area of the tank.

6. (12 pt) Suppose that f is a function such that $f'(x)>0$ and $f''(x)>0$ for all x.
   a) Explain why this function has at most one root
   b) If this function does have a root, explain why Newton's method will always produce approxmations that are too large.