MATH 165
FALL 2005
EXAM 3

1. (32 pt) Evaluate the following limits:

a) $\displaystyle\lim_{x\to\infty} \frac{x+\sin(3x)}{2x}$ b) $\displaystyle\lim_{x\to 0}(1+\tan(ax))^{\frac{b}{x}}$

c) $\displaystyle\lim_{x\to 0}(\cot(2x) - \frac{1}{2}\csc(x))$ d) $\displaystyle\lim_{x\to 1^+}(x-1)\ln(\ln(x^m)), m \neq 0$

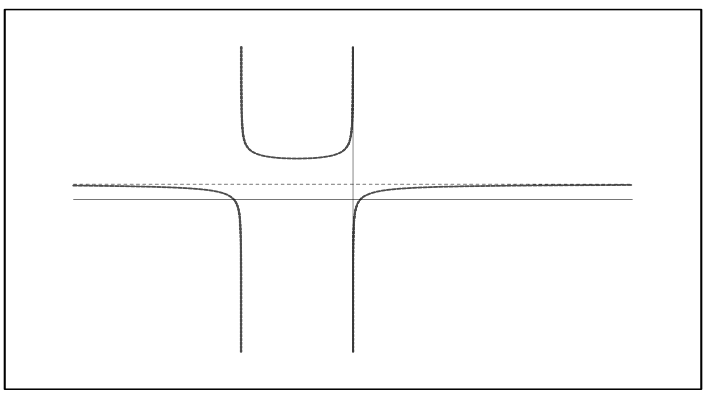
2. (16 pt) Sketch the graph of $f(x) = (x^3-3x)^{\frac{1}{3}}$. The first two derivatives are given below.

$$f'(x) = \frac{x^2-1}{(x^3-3x)^{\frac{2}{3}}}$$

and

$$f''(x) = \frac{-2x^2-2}{(x^3-3x)^{\frac{5}{3}}}.$$

3. The picture below is a graph of the derivative of the continuous function $F(x)$.



a) (6 pt) Sketch the graph of $F''(x)$.
b) (10 pt) Use this information to sketch the graph of $F(x)$ if $F(0) = 1$.

4. (18 pt) Find the largest volume of a box with square base that can be inscribed in a hemisphere. What is the proportion of this largest volume to the total volume of the hemisphere?

5. (15 pt) A window is designed in the shape of a semicircle on top of a rectangle. If we want the area to be some fixed value (say A), find the dimensions of the window that minimize the perimeter.

6. (8 pt) Suppose that $f(x)$ has the line $y = mx + c$ as a slant asymptote (in both directions) and $g(x)$ has the line $y = nx + d$ as a slant asymptote (in both directions) with $n, m \neq 0$. Find

$$\lim_{x\to\infty} \frac{f(ax)}{g(bx)}$$

where $a, b \neq 0$. You may assume that f and g are differentiable.

7. (5 pt) Carefully explain how the formula that we obtained in class for Newton's Method is derived.