MATH 165
FALL 2006
EXAM 3

1. (32 pt) Evaluate the following limits:

a) $\lim_{x\to-\infty} \dfrac{\sqrt{x^2+1}}{3x+2}$ b) $\lim_{x\to 1} \dfrac{x^3-1}{x^3-2}$,

c) $\lim_{x\to\infty} ax^2\tan^2(\dfrac{b}{x})$ d) $\lim_{x\to 0}(1+tx)^{\csc(sx)}$, $s\neq 0$.

2. (18 pt) Sketch the graph of $f(x)=\ln(|x^4-2x^2|)$. The first two derivatives of this function are

$$f'(x)=\frac{4(x^2-1)}{x(x^2-2)}$$

and

$$f''(x)=\frac{-4(x^4-x^2+2)}{x^2(x^2-2)^2}.$$

3. (16 pt) Sketch the graph of $g(x)=x^{\frac{1}{3}}(1-x^2)^{\frac{2}{3}}$. The first two derivatives of this function are

$$g'(x)=\frac{1-5x^2}{3x^{\frac{2}{3}}(1-x^2)^{\frac{1}{3}}}$$

and

$$g''(x)=\frac{10x^4-16x^2-2}{9x^{\frac{5}{3}}(1-x^2)^{\frac{4}{3}}}.$$

(Hint: The two real roots of $10x^4-16x^2-2$ are approximately 1.31 and -1.31.)

4. (18 pt) A cylindrical barrel is full of a fluid of density ρ. If the barrel is tipped so that its base is in a plane that is at an angle of θ with the ground, then the force on the bottom of the barrel is

$$F=\rho\pi R^2(R\sin(\theta)+h\cos(\theta))$$

where R is the radius of the circular base and h is the height of the barrel. What is the maximum force on the bottom of the barrel and at what angle is the bottom most likely to fail?

5. (16 pt) Let $f(x)$ be a differentiable function and suppose that the point (a,b) is not on the curve $y=f(x)$. Show that if $(c,f(c))$ is the (a) point on the curve $y=f(x)$ closest to (a,b), then the tangent line to $y=f(x)$ at $(c,f(c))$ is perpendicular to the line segment connecting (a,b) and $(c,f(c))$.

6. (10 pt) Consider the function $f(x)=x^2+b$ where $b>0$. Find a positive initial approximation for Newton's method (x_1) such that the third approximation (x_3) is the same as x_1.