MATH 165
FALL 2007
EXAM 3

1. (32 pt) Evaluate the following limits:

a) $\lim_{x\to 0} \frac{\sin(x^2) - x^2}{x^6}$ b) $\lim_{x\to 0} f(x)^{f(x)}$, where $f(0) = 0$ and $f'(x)$ is continuous at 0.

c) $\lim_{x\to \frac{\pi}{2}^-} (\tan(x) - \sec(x))$ d) $\lim_{x\to\infty} \tan^{-1}(\frac{e^x + x}{x^2 - 2e^x})$

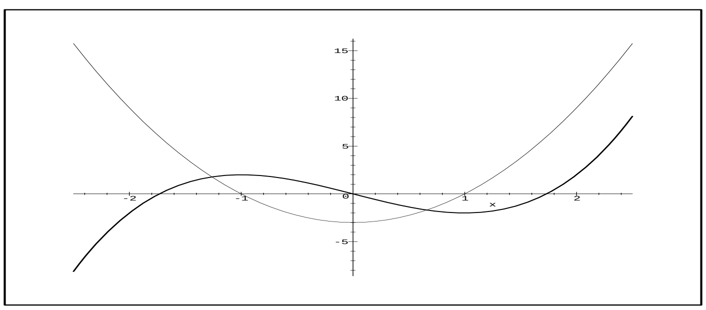
2. (32 pt) (*Two "Close" Graphs.*)

a) Sketch the graph of $f(x) = \frac{\sqrt[3]{x}}{x^2+1}$. For your convenience (and sanity) the first two derivatives are given by $f'(x) = \frac{1-5x^2}{3x^{\frac{2}{3}}(x^2+1)^2}$, $f''(x) = \frac{2(20x^4-17x^2-1)}{9x^{\frac{5}{3}}(x^2+1)^3}$.

b) Sketch the graph of $f(x) = \frac{\sqrt[3]{x}}{x^2-1}$. For your convenience (and sanity) the first two derivatives are given by $f'(x) = \frac{-1-5x^2}{3x^{\frac{2}{3}}(x^2-1)^2}$, $f''(x) = \frac{2(20x^4+17x^2-1)}{9x^{\frac{5}{3}}(x^2-1)^3}$.

Note: The two real roots of $20x^4 - 17x^2 - 1$ are approximately $-.95$ and $.95$ and the two real roots of $20x^4 + 17x^2 - 1$ are approximately $-.24$ and $.24$

3. (12 pt) Below is pictured the graph of a function, $f(x)$ (pictured darker), and its derivative, $f'(x)$ (lighter). Use this graphical information to sketch the graph of $F(x) = \ln(|f(x)|)$ (you may use only the first derivative of $F(x)$).



4. The center of mass of a can (with mass M_c) with base radius R and height H containing a liquid that is h units deep ($h \leq H$) of density ρ is given by

$$\overline{z} = \frac{M_c H + \rho\pi R^2 h^2}{2M_c + 2\rho\pi R^2 h}.$$

a) (4 pt) Give an equation that expresses the meaning of the sentence "the center of mass equals the depth of the liquid" (your equation should not contain the variable $\overline{z}$).

b) (8 pt) Show that the center of mass is minimized when it is equal to the depth of the liquid.

5. (16 pt) Find the point(s) on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ closest to the point $(1, 0)$.

6. (6 pt) Suppose that f is a function with a root r such that $f''(x) > 0$ for all x in some interval containing r. Additionally suppose that when you apply Newton's method, it converges to the root r. Will the approximations be too large, too small, or is there no way to tell? Explain.