MATH 165
FALL 2008
EXAM 3

1. (32 pt) Evaluate the following limits:

   a) $\lim_{x\to\infty} \cos(\frac{x+\sin(2x)}{x^2})$

   b) $\lim_{x\to 0}(1+f(x))^{\frac{1}{g(x)}}$, where $f'(x), g'(x)$ are continuous and $f(0) = g(0) = 0$ and $g'(x) \neq 0$.

   c) $\lim_{x\to 1} \ln(\sqrt{\frac{x^3-2x+1}{x^4+x^3-2}})$ d) $\lim_{x\to 0} \frac{e^{x^2}-1-x^2}{x^4}$

2. (16 pt) Sketch the graph of $f(x) = \ln(|\frac{x}{x+1}|)$. The first two derivatives of this function are $f'(x) = \frac{1}{x(x+1)}$ and $f''(x) = -\frac{2x+1}{x^2(x+1)^2}$.

3. (20 pt) Sketch the graph of $f(x) = (x^2-1)^{\frac{2}{3}}$. The first two derivatives of this function are $f'(x) = \frac{4x}{3(x^2-1)^{\frac{1}{3}}}$ and $f''(x) = \frac{4(x^2-3)}{9(x^2-1)^{\frac{4}{3}}}$.

4. In a very tall gallery you want to hang a mural that is h feet tall with the bottom of the mural $a > 0$ feet above eye level.

   a) (16 pt) Find the distance that you should stand from the mural to maximize your view of the mural.

   b) (4 pt) If the mural is 21 feet tall and people are going to observe from a carpet 10 feet from the wall, how high above eye level should the bottom of the mural be?

5. (16 pt) A tank is to be constructed of some fixed volume V. The tank is built by placing two hemispheres of radius R at each end of a cylinder of height h and radius R. If the cost of making the hemispherical ends is twice as much as the cost of making the cylinder (per unit area), find the ratio of h to R that minimizes the cost of the tank.

6. (6 pt) Suppose that the functions $f(x)$ and $g(x)$ both have horizontal asymptotes $y = a$ and $y = b$ respectively (towards positive infinity). Explain why the functions $f(x) + g(x)$ and $f(x)g(x)$ both have horizontal asymptotes (and find them). Is the same true for slant asymptotes? Explain why or give a counterexample.