MATH 165
FALL 2009
EXAM 3

1. (40 pt) Evaluate the following limits:

a) $\lim_{x\to 0^+} \tan^{-1}((1+\frac{2}{x})^x)$ b) $\lim_{x\to -\infty}(x+\sqrt{x^2+ax})$

c) $\lim_{x\to 0} \frac{e^{ax}-1-ax}{x^2}$ d) $\lim_{x\to 0^-} \frac{x}{\sqrt{\ln(x^2+1)}}$ You may assume that this limit exists.

2. (16 pt) Sketch the graph of $f(x)=\sqrt{\ln(x^2+1)}$. The first two derivatives of $f(x)$ are given by

$$f'(x)=\frac{x}{(x^2+1)\sqrt{\ln(x^2+1)}}$$

and

$$f''(x)=\frac{(1-x^2)\ln(x^2+1)-x^2}{(x^2+1)^2(\ln(x^2+1))^{\frac{3}{2}}}.$$

The only root of the function $(1-x^2)\ln(x^2+1)-x^2$ is $x=0$.

3. (15 pt) Sketch the graph of $f(x)=x^{\frac{1}{3}}(1-x^2)^{\frac{2}{3}}$. The first two derivatives of $f(x)$ are

$$f'(x)=\frac{1-5x^2}{3x^{\frac{2}{3}}(1-x^2)^{\frac{1}{3}}}$$

and

$$f''=\frac{2(5x^4-8x^2-1)}{9x^{\frac{5}{3}}(1-x^2)^{\frac{4}{3}}}.$$

4. (15 pt) The volume of a circular cone of base radius R and height h is given by $V=\frac{1}{3}\pi R^2 h$ and the surface area (not including the circular base) is given by $A=\pi R\sqrt{R^2+h^2}$. You want to build a conical storage tank to store a fixed volume V and you want this structure to have the least above-ground surface area (that is, you can ignore the circular bottom). Find the radius (in terms of V) that minimizes the surface area.

5. (16 pt) A fence that is h feet tall is a feet from an extremely tall building. Find the length of the shortest ladder that will reach over the fence to the building.

6. (8 pt) Suppose that $f(x)$ and $g(x)$ are differentiable and $f(0)=g(0)=0$. If $f'(x)=3(g(x))^2$ and $g'(x)=f(x)g(x)$ and you know that $\lim_{x\to 0}\frac{f(x)}{g(x)}$ exists, then what are the possible value(s) of this limit?