MATH 165
FALL 2010
EXAM 3

1. (40 pt) Evaluate the following limits:

a) $\lim_{x\to a} \frac{\tan^{-1}(xf(x))}{f(x)}$ where $\lim_{x\to a} f(x) = 0$ and $f'(x)$ is continuous and nonzero at a.

b) $\lim_{x\to 0}(Ax^2 + Bx + 1)^{\frac{c}{x}}$ c) $\lim_{x\to 0} \frac{A\cos(x^2) - A}{x^4}$ d) $\lim_{x\to\infty} \frac{\sqrt[3]{x^6 + x^2 + 2}}{3x^2 + 2}$

2. (16 pt) Sketch the graph of $f(x) = \ln|x^3 + 8|$. The first two derivatives of $f(x)$ are given by

$$f'(x) = \frac{3x^2}{x^3 + 8}$$

and

$$f''(x) = \frac{-3x(x^3 - 16)}{(x^3 + 8)^2}.$$

3. (15 pt) Sketch the graph of $f(x) = \tan^{-1}(\frac{x^2}{x^2-1})$. The first two derivatives of $f(x)$ are

$$f'(x) = \frac{-2x}{2x^4 - 2x^2 + 1}$$

and

$$f'' = \frac{2(6x^4 - 2x^2 - 1)}{(2x^4 - 2x^2 + 1)^2}$$

(the real roots of $6x^4 - 2x^2 - 1$ are approximately -.78 and .78).

4. (15 pt) You have A square inches of material from which to make a soup cup (circular cylinder with an open top). Find the ratio of the height to the radius that maximizes the volume of the cup.

5. (16 pt) A square piece of cardboard (of side length s) has a square of equal length removed from each corner so that it can be folded up into a box. Find the length of the removed squares such that the volume of the folded box is maximal (and what is the maximal volume?).

6. Suppose that $f(x)$ is differentiable everywhere and is always increasing and concave up.

a) (4 pt) Briefly explain why $f(x)$ has at most one root.

b) (4 pt) If $f(x)$ has a root and Newton's method converges to this root, will the successive approximations be too large, too small, or is there no way to tell?