MATH 165
FALL 2011
EXAM 3

1. (30 pt) Evaluate the following limits (for part c) you may assume that the limit exists and is nonzero):

a) $\lim_{x\to\infty} \tan^{-1}(\ln((1+\dfrac{2}{x})^x))$ b) $\lim_{x\to 0} \dfrac{\tan(x^3)-x^3}{x^9}$

c) $\lim_{x\to a} \dfrac{f(x)}{g(x)}$; f, g are nonnegative and differentiable with $f(a)=g(a)=0$, $f'(x)=(g(x))^2$, $g'(x)=f(x)g(x)$.

2. (16 pt) Sketch the graph of $f(x)=\frac{x}{\sqrt[3]{x^2-1}}$. The first two derivatives of $f(x)$ are given by

$$f'(x)=\frac{x^2-3}{3(x^2-1)^{\frac{4}{3}}}$$

and

$$f''(x)=\frac{-2x(x^2-9)}{9(x^2-1)^{\frac{7}{3}}}.$$

3. (15 pt) Sketch the graph of $f(x)=\cos(\frac{x}{x^2+1})$. The first two derivatives of $f(x)$ are

$$f'(x)=\frac{(x^2-1)\sin(\frac{x}{x^2+1})}{(x^2+1)^2}$$

and

$$f''=\frac{(-x^4+2x^2-1)\cos(\frac{x}{x^2+1})+(-2x^5+4x^3+6x)\sin(\frac{x}{x^2+1})}{(x^2+1)^4}$$

(the real roots of the numerator of $f''(x)$ are approximately $\pm.4$ and ±1.6).

4. (15 pt) There is a fence a feet from a very tall building. If you have a ladder of length L, find the maximum height of the fence so that the ladder will reach over the fence and touch the building.

5. (16 pt) Find the area of the largest isoceles triangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. For ease you may assume that the nonequal base is parallel to the $x-$axis.

6. (8 pt) Suppose that you are using Newton's method to find a the root of $f(x)=\sqrt[3]{x}$. Show that if x_n is the n^{th} approximation then $x_{n+2}=4x_n$. Explain why Newton's method will not work for this function.

7. (10 pt) Find the area under the curve $f(x)=x^2, 0\le x\le t$. You should get a formula in terms of t (call it $g(t)$). What is the relationship between $g(x)$ and $f(x)$?